# Child and Man

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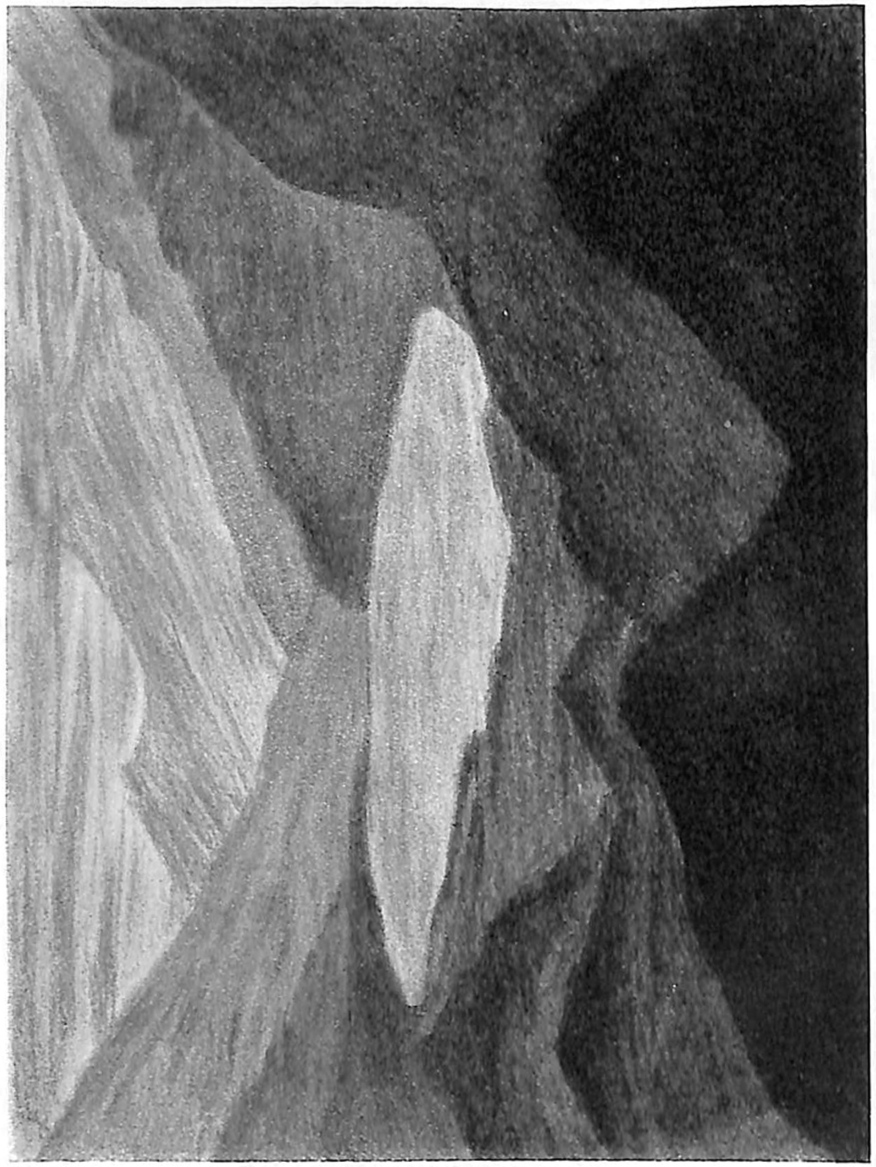

LANDSCAPE BY A BOY OF FOURTEEN

# CHILD AND MAN

*Edited by A. C. Harwood*

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## *Geography of the Earth*

At the beginning of a geography course with older children, even if one wishes to make a detailed study of one country only, it is well to make a start with those facts which are connected with the world as a whole, and with the relation of the Earth to forces outside the Earth itself. There are two main reasons for this. For one thing, the rule of passing from the general to the particular is almost as much a practical necessity in the study of geography as it is in physiology; for one cannot attempt to explain differences of climate, geological formation, and so on, over any portion of the Earth's surface, except by considering it as a part of a great organism, any more than one can describe the nerve endings and blood circulation in one's finger without describing how the finger is connected with the heart and spinal column. With children of fifteen and sixteen there is the additional necessity of filling their thoughts and feelings with the larger aspects of all the subjects they study, for when puberty has been reached there can grow in them an impulse of love for the whole world. Yet there may also arise at this time a tendency to be preoccupied with their own personal concerns and difficulties, and this tendency can be counteracted by leading their thoughts to a wide view of life.

It seems natural to start a geography course with a consideration of some aspects of the Earth's relation to the Sun, and a study of the Solstices and Equinoxes helps one to realize how far-reaching are the effects of a variation of this relationship. It also helps one to feel the Earth's hunger for the light and warmth of the Sun; for does not the Earth, in its movements, strive towards the Sun, just as each separate plant growing on its surface strives towards the light? The Earth brings it about by its rhythmical movements that each portion of its surface is turned towards the Sun in the course of a Solar Year. A subsequent study of phenomena of climate and weather; of geological conditions; of differences of vegetation and so on in various parts of the globe brings it home to the children even more forcibly that the Earth cannot be considered except in its relation to other cosmic bodies, for they find that winds, ocean currents, rivers, even glaciers owe their origin to the Sun.

Comparisons of the Earth organism to the human organism come readily to mind, for the two are composed of similar substances; in fact, the human body can in a sense be looked upon as a very finely developed portion of the Earth. The solid element in our bodies, the bony skeleton, has gradually hardened out of softer and more plastic substances, in the same way that the solid element in the Earth has gradually hardened out of something softer. The four elements of which the ancients taught—Fire, Air, Water, and Earth—are all found in our bodies as in the outer world, and in neither case does the solid element predominate. There is much more fluid in our bodies than solid matter, and much more ocean than land on the Earth's surface.

The Greek philosopher, Empedokles, spoke of these four elements somewhat in this way: "The Earth has a solid crust which is largely covered by a watery sphere. This again is surrounded by a sphere of air, and at the limits of the atmosphere is a mantle of warmth. The three conditions of Water, Air, and Fire are found again in the interior of the Earth, but there is only one manifestation of the solid element, this occurs once only". In this connection it is very interesting to find that recent investigations in the upper atmosphere have shown that, whereas up to a certain height, about 10 kilometres, the temperature becomes lower the higher one ascends, yet above this height it increases again.

We can distinguish a threefold organization of the Earth's

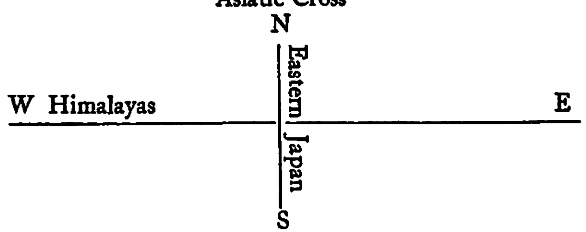


surface, each zone being differently related to the Sun. Man's body is composed of three systems, which serve his three activities of Thinking, Feeling, and Willing. Man's body has a Head and Nerve System, a Limb and Metabolic System, and a Rhythmic System (heart and lungs). The temperature in the various organs of the body is not uniform, but the coolest part is the head, and the hottest the digestive organs, specially the liver. The cooler blood coming from the head mingles, however, with the warmer blood coming from the Metabolic System which it meets in the heart. In the cool head there is a state of calm which can be compared with the rigidity of the Poles. At the Poles the movement of the Earth turning on its axis comes to rest. In our Metabolic System there is movement and heat. In the Torrid Zone the heat is the greatest on the Earth owing to the fact that the Sun is directly overhead. It is at the Equator, too, that the Earth revolves with the greatest speed. The Temperate Zones stand midway between these extremes of temperature and movement, and in a similar way the Rhythmic System has neither the turmoil of the digestive processes nor the immobility of the head, but it has instead its own rhythmic breathing and heart beat.

It is a great satisfaction to the children when studying a map of the world to discover that the distribution of land and water as at present arranged on the surface of the Earth has a definite plan, and is not chaotic as it appears to be at first sight. The fact of a great land mass in the northern hemisphere corresponding to a great region covered by water in the southern hemisphere, and vice versa, is particularly clearly brought out when covering a globe with a map in relief, as was done by one of the classes of the New School last year. The Antarctic Continent, for instance, is anti-podal to the Arctic Ocean; Europe and Africa to the Central Pacific; North America to the Indian Southern Ocean; and the northern land mass of South America to the West Pacific.

Another fact of deep significance for everyone on the Earth can be brought before the older children in their study of geography. I refer to the relation between certain forces that have shaped the Earth's crust, and are also related to man's gradual development of consciousness. Modern geologists speak of a mountain or a range of mountains as having its strata laid down in one or another direction, for instance North—South. This means that instead of confining oneself to examining the *substances*, one endeavours to glean from them an idea of the *forces* that have been at work in shaping the Earth's crust. Speaking generally, it will be found that the large mountain ranges of Europe and Asia are on a line running parallel with the Equator, that is, East—West, whereas the mountain ranges of the New World are on a line running North—South. Apart from this general plan, however, there are places on the Earth where a concentration of forces working in different directions is evident. For instance, the mountains of Eastern Japan are in a North—South direction, and the Himalayas stretch from East to West. Let us try to picture to ourselves the tremendous forces that must have been active in order to make those mountain folds go in one particular direction. In Central Asia we can imagine the forces which shaped the mountains of Japan intersecting the forces which formed the Himalayas, thus:*

Asiatic Cross




and when we seek the focus of these forces we find that it is in Tibet, whence arose the first great human culture after the Atlantean catastrophe, the Ancient Indian Civilization. One can go further, and show that in Europe, which has been chosen for the development of our modern consciousness, another cross is inscribed by the forces which shape the Earth's crust and continue to work in its atmosphere. By studying the material substances of the Earth we can realize that there is a harmony between these formative forces and the development of human consciousness. After the great changes had occurred on the Earth which led to the disappear-

* The diagram is taken from the article by Dr. Guenther Wachsmuth "Erden-Anslitz und Menschen Schicksal," published in *Gaia Sophia*, 1926.



ance of the continent of Atlantis, the survivors were brought by their leaders to Central Asia to found there a new culture. This spot was chosen because the leaders knew that exceptional structural forces were at work there. The Persian civilization followed when the Ancient Indian culture had fulfilled its task, and the civilization which followed this, the Egyptian, was still further to the west. After the Egypto-Chaldean epoch came the Greco-Roman period of culture, and in modern times the re-discovery and opening up of the American Continent has carried civilization still further to the west. Successive cultural periods since the Atlantean catastrophe, therefore, show a progression from East to West.

Such considerations as these should lead to a deep feeling of kinship between man and the Earth on which he is active between birth and death. The children can be reminded, also of a fact with which they are familiar through their study of physiology, namely, that the same rhythmic law that works in the relation of the Earth to the Sun and Stars works also in man's breathing.*

There is another side to the study of geography, the importance of which can hardly be overrated, and that is the realization of how much we owe to the activities of far-away people for the comforts, even the necessities, of our daily lives. A visit to the docks and wharves helps to enliven the study of imports and exports, and is full of interest besides. It should become clear to the children as they study geography that man in modern times has extended his knowledge of the world enormously, and that through modern facilities of communication no country can be unaffected by the doings of its neighbours. Something of this idea was expressed in a recent play: A man is reproached by a friend for his lack of interest in his neighbours, at which he exclaims: "But what have I in common with these people?" The friend, exasperated, retorts, "Why, man, have you not the whole Earth in common with them?"

Reference has been made to the fact that the principal mountain ranges of the Old World follow a line East—West., while those in the New World are North—South. But not only do the forces at work form different structures in the mineral

* See the article "First Lessons in Physiology," in Vol. I, No. 3 of this magazine.

substance of the Earth, but the characters of the peoples inhabiting these parts of the Earth are different. We can characterize the people of the East by saying that they seem more aware of the Cosmos and its eternal laws, whereas the American is keenly alive to the Earth and its individualizing influence. The mountains which lie East—West all come into the same relation with the Sun during the course of the day, but those lying North—South each have a different relationship with the Sun, and, as we have seen, the fact of paramount importance for any portion of the Earth is its relation to the Sun.

G. H. SARGEANT.

## *Charcoal Drawing*

### *What does it mean to a Child at Puberty?*

THE plastic and pictorial arts play an integral part in schools working on the methods of Rudolf Steiner. It is obvious that when the child's power of imagination has first been stimulated and awakened in the various subjects he learns, an outlet and an expression in an artistic form must be found for the strong forces of the Will which imagination always produces.

All artistic activity gives opportunity for the subconscious life of the child to express itself and therefore to become objective and to find its right form. For in art, as all great men of the past have experienced it, man transcends his mere physical surroundings to the idea underlying creation, to the picture of beauty and truth. Thus the Greeks created perfect human figures, not as copies of nature but as ideal forms, and experienced their Gods in the forms they created.

Our age is different from that of the Greeks in so far as our intellect has become adjusted to our awakened powers of sense perception. What we see with our eyes we can retain in memory; or we can express it by drawing, provided we have a keen sense of observation. This is not yet art, however, for art must be original; it must proceed from deeper layers of human consciousness than the enumerating analytical intelligence that proceeds from the brain. These deeper layers are in the sphere of imagination which does not consist purely of



entities derived from the visible world. Creative imagination has for ever been the source of those works of painting, music, poetry, and art generally which to this day awaken in man the highest ideals, awaken in him the consciousness of the divine origin and destiny of his soul.

Now at one particular stage in his development, a child tends to feel himself estranged from the world by forces which surge up from the depths of his own being, and which he cannot explain. He becomes intensely wrapped up in his own problems, he appears to be in a maze into which no light shines. He is often in a mood of despair, and if no light comes to illumine this darkness he begins to lose faith in human nature.

This age is the age of puberty, and when a boy or a girl comes into this age—it is the threshold from childhood to manhood which everyone must cross—we can best assist this struggle by bringing it into the sphere of art.

No artistic medium can so well express the conflicting forces of this age as black and white. At the age of puberty, then, we put aside painting and take to charcoal, which lends itself to strong form and movement, and develops a strong dramatic element. At once the scene is changed; what before was bright sunshine, shedding glory on legend and fairy-tale, now becomes a thunderstorm with threatening clouds illumined only by a dazzling streak of lightning. From placid serenity the sea turns into a furious cavalcade of waves dashing their angry heads against the rocks. The gentle slopes of hills are now gaunt precipices, or lonely ice-bound mountain crags. But we do not stop at this grim aspect of nature. After the thunderstorms the sky will be clearer, the air fresher, and the sunshine brighter. Now is the battle won: next time we shall be stronger.

Sometimes the child will introduce the strongest contrasts of light and darkness into one picture, and then curious and most imaginative pictures of this subconscious battle taking place in the soul at the time of puberty may be observed, as, for instance, in the illustration facing page 8.

This is a striking example of the entirely unaided and uninfluenced work of a child of 14, and for this reason aptly illustrates what is taking place. The half recumbent figure at the bottom of the picture, and the one above it and to the left, can be imagined as the child herself in two stages of her development or perhaps in two moods (note how the upper figure is less dark).

On either side of the figures there are others clearly expressing their qualities. The one on the left (seen from the observer) is pictured as an angel praying for the salvation of this striving soul; on the right a hideous, misshapen creature, "hobgoblin or foul fiend," in the gesture of dismay at having lost this soul. But none of this is stationary, least of all the two figures, although half recumbent; for they are in the centre of a rushing sweep of upward streaming light, which leaves no one doubting who is the victor and who the vanquished.

For the teacher it is gratifying to experience the joy of a child who has thus fought a battle and won it; for he must take an active part in the struggle, continually encouraging when hearts begin to fail, quietly suggesting a new idea when he hears cries of "I can't go on," and for ever striving to be the example in strength and courage and perseverance in the battle. No mere skill in drawing avails; it is a question of surrendering one's own ideas and uniting oneself with the problem of the individual child. In this way art in education means healing—the healing which again means hallowing.

A. W. Kaufman

## *The Imaginative Treatment of Geometry in Education.**

Anyone who looks at the life and work of Rudolf Steiner, even in a superficial way, cannot fail to be deeply impressed by the scope and comprehensiveness of his teaching. It is not an exaggeration to say that there is no sphere of human knowledge, experience, or achievement with which he has not dealt, at times entering into great detail and at others giving only general indications and then leaving his pupils to develop further what he has said. But in this very comprehensiveness there lies a certain danger. If one separ-

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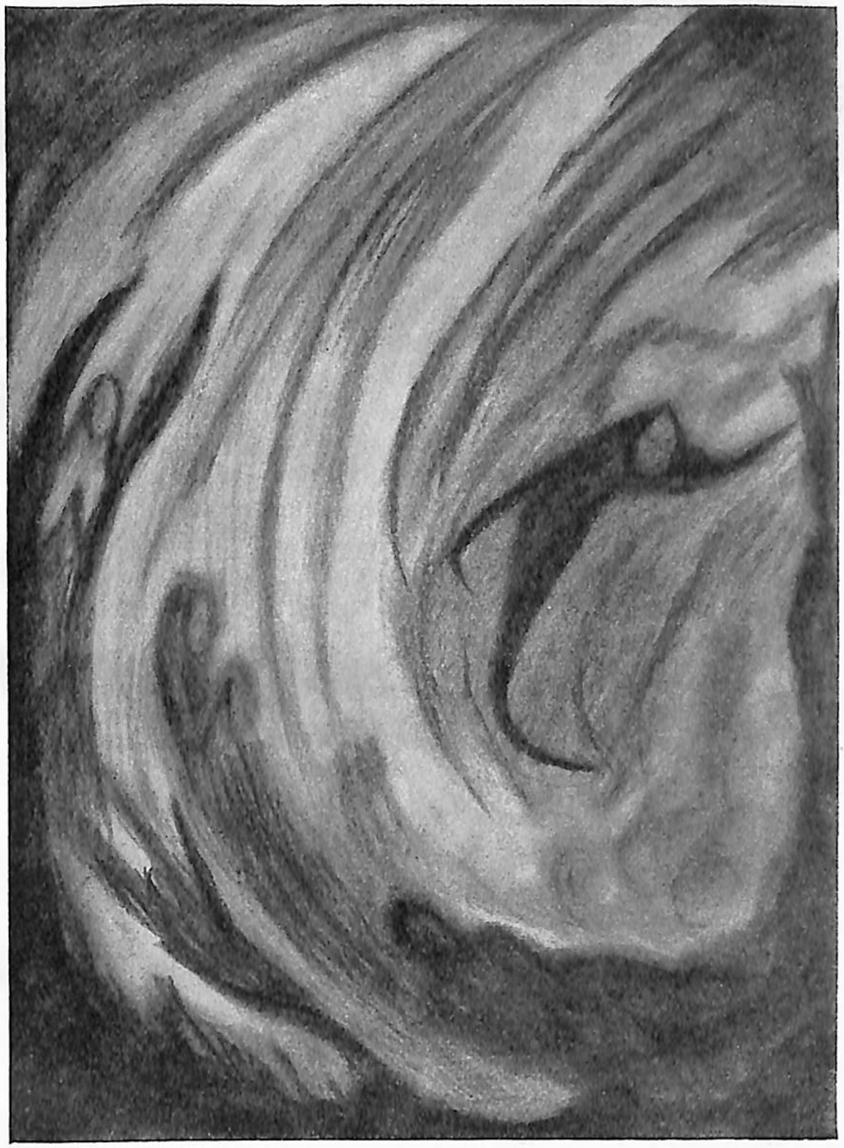


BY A GIRL OF FOURTEEN

ates any part of an organism from the whole, that part ceases to live and the vitality of the whole organism is impaired. This is a self-evident truth. One may think to apply certain aspects of Rudolf Steiner's teaching in a sphere in which one is perhaps particularly interested, without entering deeply into the fundamentals of his work. If such a procedure is even attempted, it is bound to fail. This body of teaching is an organic whole and the life-force of Spiritual Science which Rudolf Steiner has given to the world is flowing through every part of this organism. Whoever wishes to study with seriousness what Anthroposophy has to say on any particular subject, cannot do so without being led directly to the very centre of Anthroposophical teaching—to the answer to the question: What is man?

The truth of what has been said may be seen very clearly in the sphere of Education. For here, the impulse given by Rudolf Steiner is being worked out *practically* in schools on the Continent and in England. A teacher cannot take a *part* of these educational ideals and apply them with any success. He must take the whole. And where will this lead him? It will lead him deeply into the fundamental truths of Spiritual Science.

"A true pedagogy must be based upon a knowledge that embraces man with respect to body, soul and Spirit. Intellectualism only grasps man with respect to his body, for to observation and experiment the bodily alone is revealed. Before a true pedagogy can be founded, a true knowledge of man is necessary. This Anthroposophy seeks to attain."

It is not within the scope of the present article to deal adequately with this "true knowledge of man." For this the reader is referred directly to the works of Rudolf Steiner himself. It is our purpose here to consider certain aspects of teaching, particularly in the sphere of mathematics, and, further, to show how such a subject in educational practice is organically related to other subjects and to the forces and capacities developing in the child.

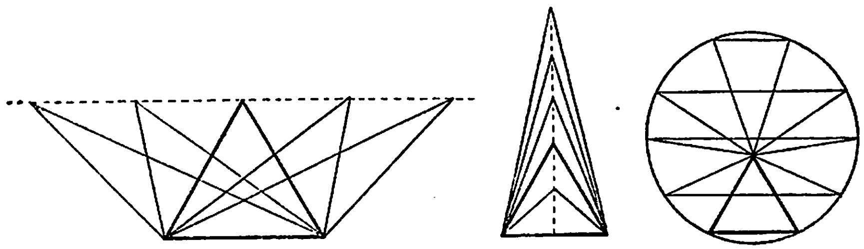
Let us turn to the Curriculum or "Teaching Plan" as given by Rudolf Steiner when the Waldorf School was founded in Stuttgart in 1919. One might read this through carefully and say: "Yes, this seems to be a very comprehensive curriculum, and children working in accordance with it should receive a wide outlook and culture." Certain differences from the general course of school work would be noted and perhaps questioned, but no one could fail to be favourably impressed by the scope of the plan. Such criticism, however, is only superficial and fails to grasp its real nature and essence. This is no mere abstract plan. There is nothing arbitrary in its arrangement. It is based, not on any "theory" of education, but on a true knowledge and understanding of the being of the child in its threefold aspect of body, soul and Spirit. It contains, hidden within it, all the mysteries of human life as these unfold themselves in the growing boy and girl. If one could read this "Teaching Plan" with a spiritually awakened eye, one would be reading there of the developing nature of the child. It is a living, organic structure, corresponding in all its details to the living being of the child. And as such it must be studied if we wish to gain a true realization of its value. Thus it is useless to regard it purely from an intellectual point of view. From such a view-point, the criticism already referred to would arise. We can only penetrate into its real nature and meaning if we read it in the light of the knowledge we gain through Spiritual Science concerning the forces and capacities which unfold in the child as he grows to manhood. Only then are we able really to draw from the wisdom which is contained within it, for then, in our teaching practice we know what we are doing and, more important still, *why* we are doing it. We recognize that this aspect of a certain subject is the right one for children of such and such an age; and we know why this is so and what is the connection with the other subjects.

From this point of view we will consider in some detail the teaching of Geometry, which begins in the Sixth Class, and illustrate with examples taken from teaching practice.

Unconsciously, in the movements of his limbs and of his whole body the child is constantly living right into the very laws of geometrical form. The teacher has therefore to help him to recognize consciously, and understand these forms; but it is the forms *in movement* which he needs at this age; everything should be capable of movement and metamorphosis. We can point to many examples in outer Nature where geometrical forms are always moving and changing; in fact, the finished



form is very often the result of the motion of some object: the curves traced out by the stars and planets in their courses; shadows cast by the sun, now growing, now diminishing and giving every variety of shape; the beautiful curve which can be seen against the dark background of the tea when the sun is shining on to the tea-cup; the path of a ball when it is thrown into the air; the ripples on a pond, caused by dropping a stone into the water, spreading in ever-widening circles; the varying figures given by the cone of light from a motor-car lamp as it shines directly on to a wall or obliquely down to the ground. These are only a few of the innumerable moving, changing geometrical forms which are a matter of everyday experience if we will only observe them; and it is from this point of view that we must bring Geometry before the chil-



(*a*) (*b*) (*c*)

FIG. I

dren from the twelfth to the fourteenth year. The magic key-words to open the door to this new sphere of knowledge are movement, change, metamorphosis. And if these keys are used, then Geometry can become something full of life for the child and not just a matter of dull, abstract, fixed forms.

The geometry of the triangle is the first thing to bring before the children, and the teacher should at once allow them to draw many different kinds in various positions. They may start with three points and join these points by straight lines, and they can also form the triangle by drawing three intersecting lines; thus at once they will understand the triangle as a figure having three angular points and three sides. The triangle will become living for the child if he draws "families" of triangles, one evolving from the other. The drawings in Fig. I show three different methods of treatment.

The teacher can develop many different ways of "evolving" the triangle; but it is important to notice that the starting-point should be the equilateral triangle, for this is the prototype of all triangles. It is the artistic form, the perfect triangle. These drawings may well be done in coloured pencils, for this gives additional joy to the children. So in this simple way we can bring this principle of movement and change into the very first lessons on the triangle, and at the same time the children will learn the differences of one form from another and gain an insight into the different properties. Such an understanding which they have experienced by observation and actual drawing is the right basis for the intellectual understanding which they will gain from the theorems and propositions after they have reached the age of fourteen. The two are, as it were, complementary. Then again, the drawing of triangles to measurement may be given at the same time; this demands accuracy and the children will also come to realize the sets of conditions necessary to enable them to draw particular triangles. The method of procedure outlined here also applies to other elementary geometrical figures.

The children must also be brought to realize that Geometry does not only deal with surfaces, with plane figures, but also with space in all directions. The teacher must give them the feeling for three dimensional space and how they themselves are living and moving in space. Thus the children's eyes will be lifted from the flat circle on the paper before them to the great spherical dome of the heavens and they will come to recognize more and more how they belong to the heights and the depths and the wide expanses. When they have this larger and more intimate understanding they can then begin to make simple models out of cardboard: first, perhaps, a little model of a house or of the classroom. Then they will see from the outside the space form with which they are familiar from the inside. Following on this they may make the five regular solids—tetrahedron, cube, octahedron, duodecahedron, icosahedron—cylinders, cones, etc. Again the construction of these solid figures will show them the necessity for accuracy, and once more they will recognize the geometrical forms of the different facets and how they are equal to one another or how they vary in their form.



The following are two illustrations of how geometrical truths can be presented to the children at this time so that they can actually see and experience the truth of what they will later prove from an intellectual point of view. (Here again we have the two complementary aspects to which reference has already been made.)

1. The three angles of a triangle are together equal to two right angles (or 180°).

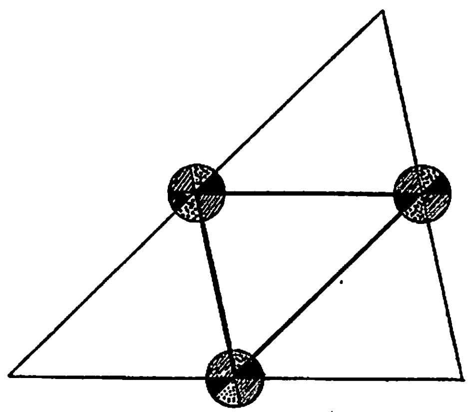


FIG. 2

This drawing explains itself. It is clear that each of the small, shaded circles contains each of the three angles of the triangle twice. The three angles together must therefore be equal to half of the right angles comprised in the whole circle, that is, to two right angles. Previously, of course, the children will have learnt angular measurement. Incidentally one can point to many other geometrical truths contained in this figure—properties of parallel lines, etc.

2. The square on the hypoteneuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

This, the well-known theorem of Pythagoras, is perhaps the most important of all geometrical truths and one which the child must grasp and experience with his whole being. The ordinary logical proof, as required for example in examinations, is highly abstract and depends on many former proofs which must first be mastered. If the child learns such a proof

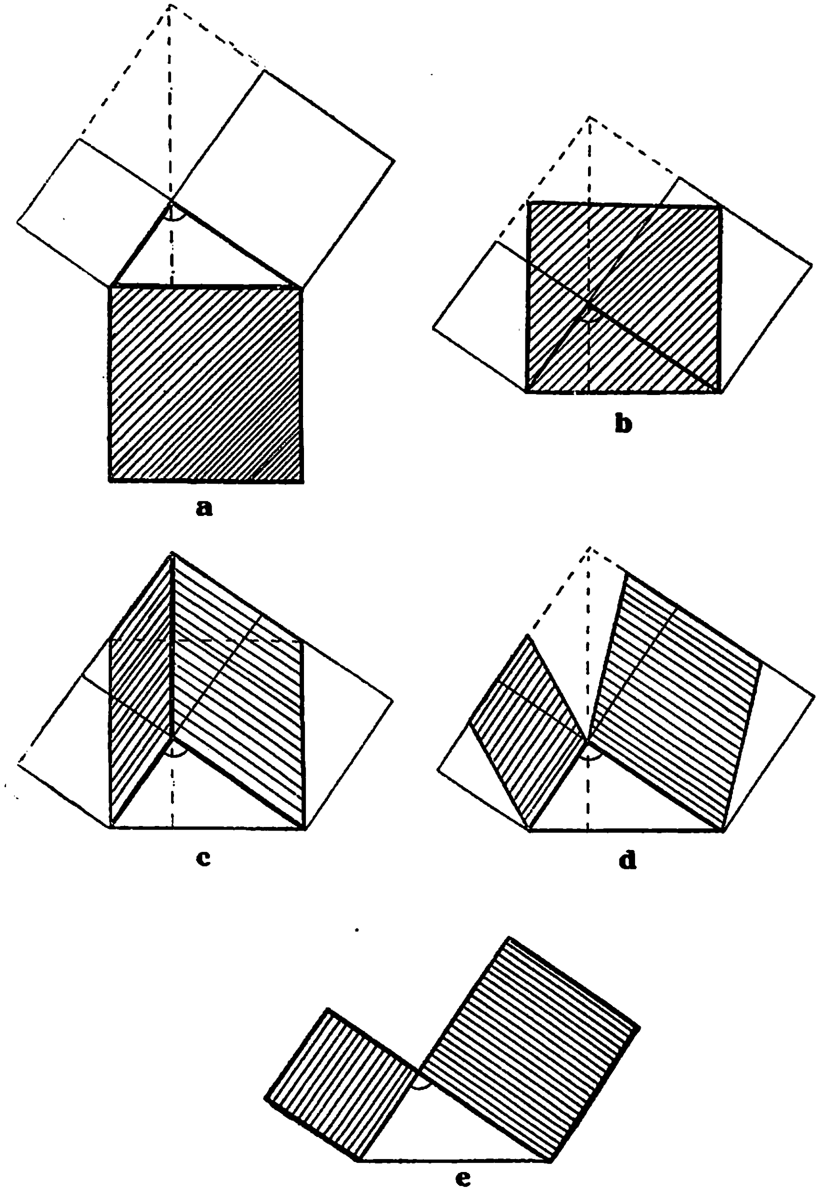



Fig. 3



only he will be intellectually convinced of the truth of this theorem, but that is all. It will be a matter of the head alone. How much greater will his understanding and conviction be if he has previously seen and felt its truth, if he can really picture the largest square as being made up of the two smaller ones. There are many ways of illustrating this geometrical fact, most of which depend upon cutting up the smaller square along certain lines and rearranging the parts to fit exactly into the large square like a jig-saw puzzle. Such a procedure *is* a puzzle and cannot give to the child the clear picture obtained by the following method worked out by Dr. von Baravalle of the Waldorf School, Stuttgart, which again depends on the principle of movement. Each of these drawings represents successive stages in the process.

Drawing (a), Fig. 3 (see p. 14), shows a right-angled triangle with the squares drawn on the sides and also the necessary lines of construction; the vertical, dotted line which is drawn from the topmost point perpendicular to the base of the triangle is also found to pass through its vertex. We have now to show that the shaded square is equal in area to the combined areas of the other two squares. First the large square is moved vertically upwards so as to stand on the base of the triangle instead of hanging from it (b). Again, in drawing this second position the children will find that the two upper angular points exactly come on the sides (or sides produced) of the two smaller squares as shown in (b). Now one must regard the two horizontal sides of the large square as elastic, and these are pulled upwards to give the next stage in the process (c). Here there has been no alteration of the area; what has been taken away at the bottom has been added at the top. This shaded figure now consists of two parallelograms and in (d) we see these two parallelogram opening out from the vertical line of construction and sliding back until they exactly coincide with the two smaller squares (e). And so the large, shaded square is by this process of movement shown to be equal in area to the two smaller squares and thereby the truth of the theorem of Pythagoras has been demonstrated. It should be added that, before this, the children have been shown, again in a pictorial way, that a parallelogram and a rectangle (or square) on the same base and between the same parallels are equal in area. Without this, one would not be justified in sliding the two parallelograms back over the two squares.) Such a "proof" as this gives to the child a vivid picture and he can easily review in memory the stages in this transformation, which can almost be compared to the shooting upwards of a plant, followed by the opening out of the blossom. The children will feel and see the reality of this great truth and they will then be rightly prepared for understanding the logical proof when they are a few years older.

The next drawing to which we will refer will give the children, perhaps better than anything else, this feeling for

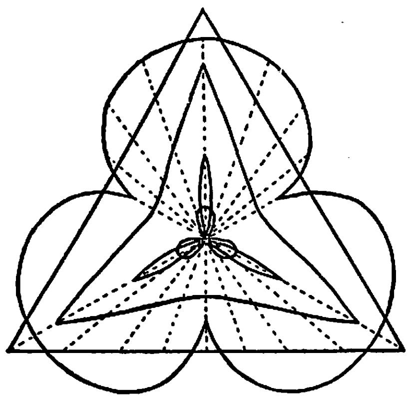


FIG. 4

the movement and change of geometrical form. In this metamorphosis we see how even a straight line becomes changed to a curved line, and how out of a triangle one can evolve very beautiful forms. As before, one starts from an equilateral triangle (Fig. 4); its centre point is found and rays are drawn from points in the sides through the centre. (To avoid confusing the figure, rays are shown drawn from points in the lower side of the triangle only.)

Now one moves each of these points in the sides the same distance along its own ray towards the centre and joins the points thus obtained by a smooth curve. Each of the sides then becomes a conchoidal curve, the whole giving rise to the maple leaf form. If this process of movement is continued



further, we get looped curves, giving the double leaf form in the centre and finally the rosette form when the points have been taken through the centre. And so by this simple process one arrives at these beautiful figures which so closely resemble leaf forms. It would even appear in this metamorphosis as though one were passing over from the mineral world with its hard, straight outlines to the world of the plants with

*Addition.*

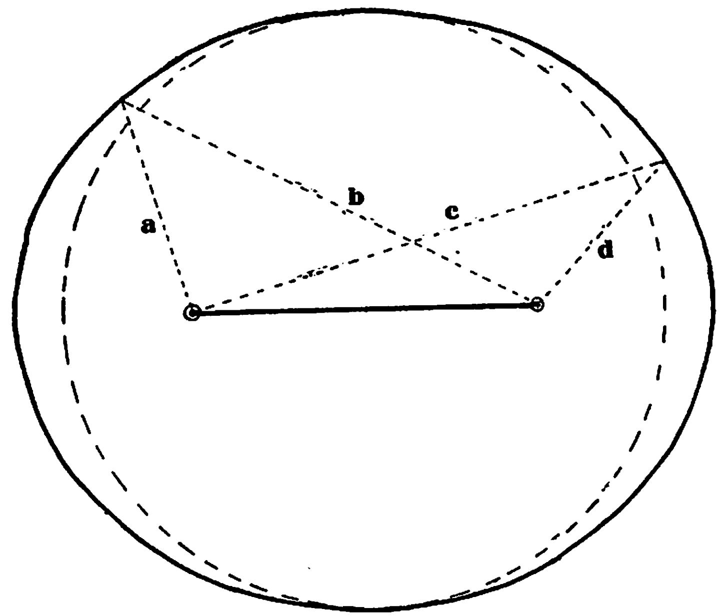



FIG. 5 (i)

their manifold curves. Other simple geometrical figures can be treated in a similar manner and the children experience keen delight in seeing how the curves move and change and evolve one from another, especially if they draw them in colour.

As a last example of this imaginative, pictorial treatment of Geometry we will consider a series of drawings which the children may do when they are a little older; they may well be introduced after the fourteenth year, accompanying the more intellectual treatment of the subject. These are a series of curves which represent pictorially the first four rules of Arithmetic—addition, subtraction, multiplication and division. In our previous examples we have been dealing with the triangle; now we come to quite different geometrical forms. Again it is good that these drawings should be carried out in colour to add to the artistic effect. We will take the four sets of curves and consider each set separately.

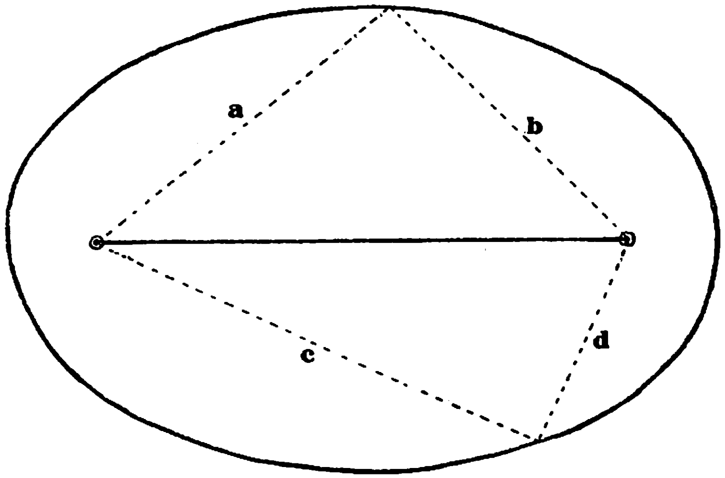



FIG. 5 (ii)

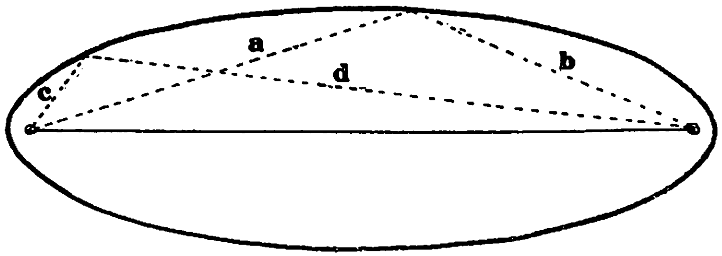



FIG. 5 (iii)

*Addition.*

A certain convenient length (say 8″) is chosen as the "constant sum," and remains constant for this set of drawings. Two points, called foci, are taken quite close together (say 4″ apart)*—Fig. 5 (i)—which remain fixed for the particular drawing (see p. 17). Now a point is found such that the

* The measurements in inches apply to the original drawings from which the drawings here are reproduced.



*sum* of its distances from the two foci is equal to the constant sum (8″). (This is done by "stepping-off" these focal distances from the two foci with compasses.) This process is repeated until a whole series of points is obtained and these are then joined by a smooth curve. Thus the sum of the focal distances of any point on this curve is a constant (8″).

$$a + b = c + d$$

This closed curve, which is a geometrical picture of *addition*, is an Ellipse. Now what happens when the foci are moved further apart (the constant sum, 8″, remaining the same)? As may be seen from the drawings, the ellipse becomes thinner and it is clear that there are definite limits. When the foci are coincident we have a circle (radius 4″), and then as they are separated we get ellipses becoming ever thinner and thinner until when the foci are a certain distance apart (in the case we are considering, 8″) the ellipse has disappeared into a straight line. And so in this process of movement we have passed from a circle, through the Ellipse, to a straight line.

*Subtraction.*

Again we choose any convenient length as the "constant difference" (e.g. 2″) and take two focal points. Now it is a question of finding all points, the *difference* of whose distances from the two foci is always the same, i.e. equal to the "constant difference." The construction is carried out as before, but the foci must be a certain minimum distance apart before any figure can be obtained. This minimum distance is equal to the "constant difference" (here, 2″). The curve obtained when this distance is greater than 2″ is the Hyperbola [Fig. 6 (i), (ii), (iii)] whose two branches open out more and more as the foci are moved further apart. Thus a series of Hyperbolas is obtained and the difference of the focal distances of any point on these curves is a constant (2″):

$$a - b = c - d$$

As may be seen from the drawings, the two branches of the Hyperbola will close up into two straight lines going out in opposite directions from the two foci; this limit will occur when the foci are a distance apart equal to the "constant difference" (2″). The other limit is when the foci are an infinite distance apart; then the branches of the Hyperbola have opened out to two parallel straight lines. Hence in this movement of the foci we have passed from two straight lines, through the Hyperbola, to two parallel straight lines at right angles to the direction of the first pair. It should further be noticed that the two poles of the Hyperbola (i.e. the two points where the branches cut the line joining the foci) do not move and that their distance apart is equal to the "constant difference."

In the case of the Ellipse, the functions of the two foci are the same, since for addition the law of commutation is valid, i.e. $a + b = b + a$. Thus the geometrical picture of addi-

*Subtraction.*

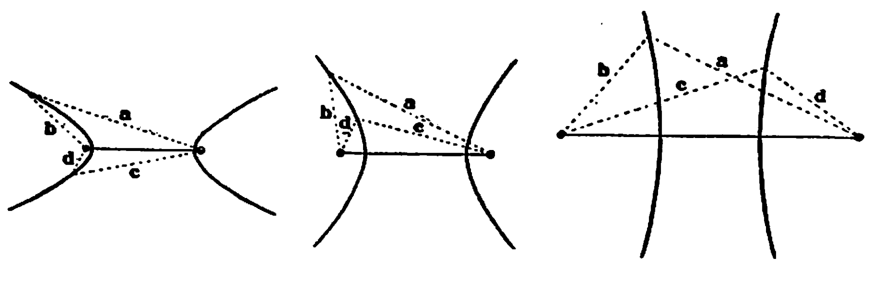



Fig. 6

tion is a closed curve. But since in subtraction this law does not hold (i.e. $a - b$ does not equal $b - a$), the functions of the two foci of the Hyperbola are different. In the construction of the Hyperbola one has to reverse the process of subtraction. Thus the reality only lies in one branch of curve and the other branch is, as it were, the image or mirror-picture. This is also always the case when a hyperbolic curve is obtained as the expression of a physical law of Nature (e.g. Boyle's law for gases).

*Multiplication.*

In the geometrical pictures of the process of multiplication we get a wonderful metamorphosis taking place (Fig. 7). The length chosen this time represents the "constant product"



(say 16″) and again two focal points are taken quite close together. By the same constructional method as before, points are found, the *product* of whose distances from the two foci is always the same, i.e. equal to the "constant product" (16″).

*Multiplication.*

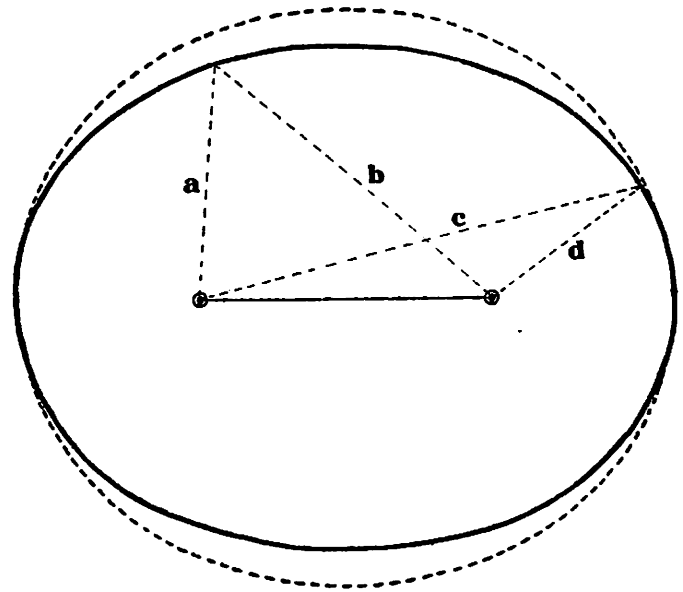



Fig. 7 (i)

These points are joined by a smooth curve. Then the product of the focal distances of any point on this curve is a constant (16″):

$$a \times b = c \times d.$$

When the foci are quite close together (say 4″ apart) an oval is obtained. This is not an Ellipse. The dotted-line curve is an Ellipse having the same foci as the oval [Fig. 7 (i)]. On moving the foci apart this oval curve becomes flattened until in one position a "flat oval" is obtained [Fig. 7 (ii) ]. Continuing the movement further the flat sides of the oval become concave (iii) until, when the foci are a certain distance apart, the figure has changed into a Lemniscate (iv). This figure arises when the distance between the foci is equal to twice the square root of the constant product. (Constant product $= 16, \sqrt{16} = 4, 4'' + 4'' = 8''$.) If the foci are now further separated by only the smallest amount, two distinct egg-shaped ovals are obtained (v) which become smaller, more rounded, and further apart, as the distance between the foci is increased

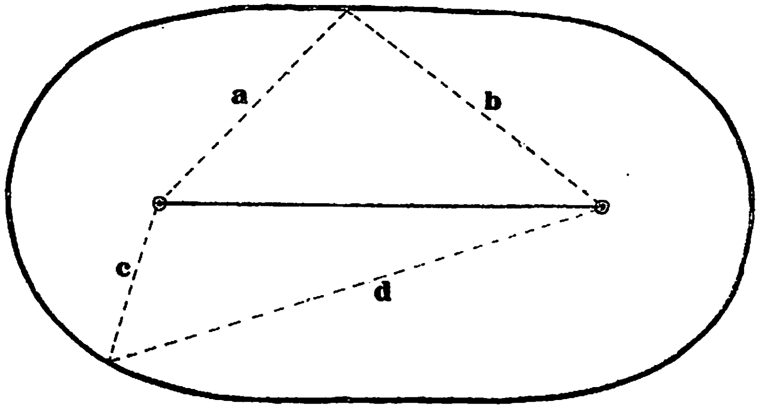



Fig. 7 (ii)

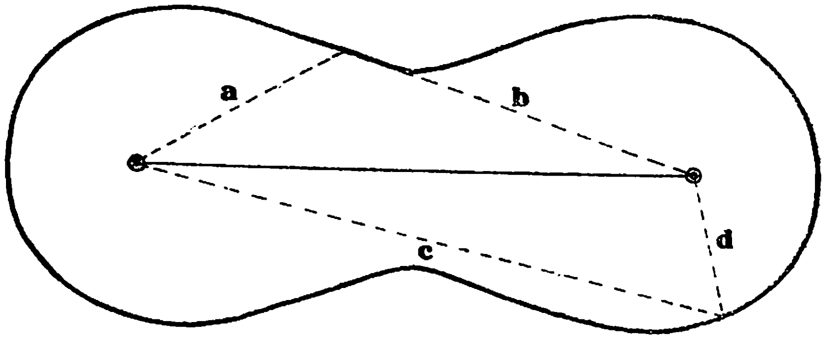



Fig. 7 (iii)

(vi). This series of curves is named after the French astronomer, Cassini.

By this geometrical process we thus arrive at two distinct oval curves which have arisen from a single oval, and we have traced the various stages by which this transformation comes about. A mere glance at these drawings will remind one at once of photographs or diagrams in books on Biology, showing the process of cell division—or rather of cell multiplication.



These geometrical figures are a picture of the natural phenomenon of the multiplication of one germ-cell into two such organisms. The comparison is very striking, and shows us

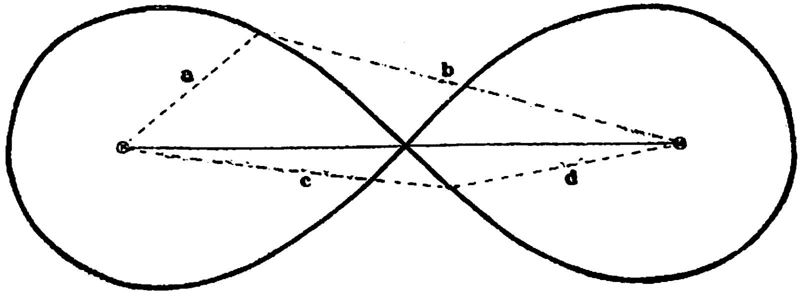



Fig. 7 (iv)

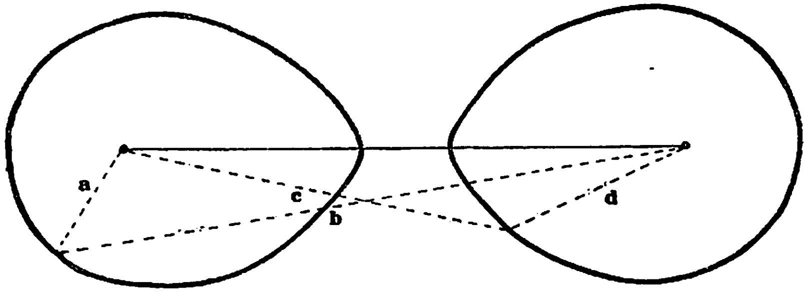



Fig. 7 (v)

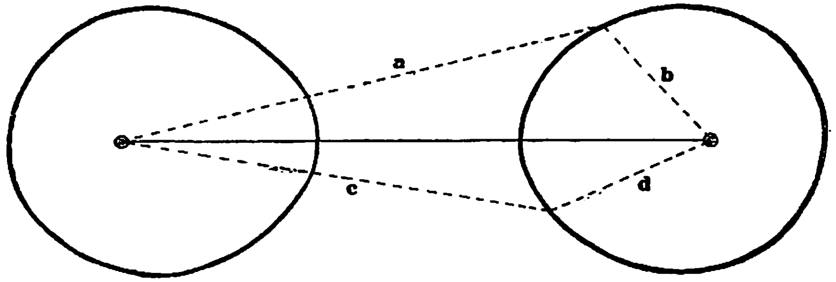



Fig. 7 (vi)

how Geometry, by being treated in a living way, may be brought into relation even with organic processes in Nature, with the living forms of Nature which are constantly undergoing metamorphosis as they grow.

In these six drawings, the product (16″) has been kept constant throughout, but the foci have been moved further and further apart. Now we may combine them all into one picture if we keep the foci fixed and vary the product (Fig. 8). Then each curve is a picture of the process of multiplication, the product becoming less as we pass from the large oval, through the flat oval to the Lemniscate and on to the separated egg-shaped ovals:

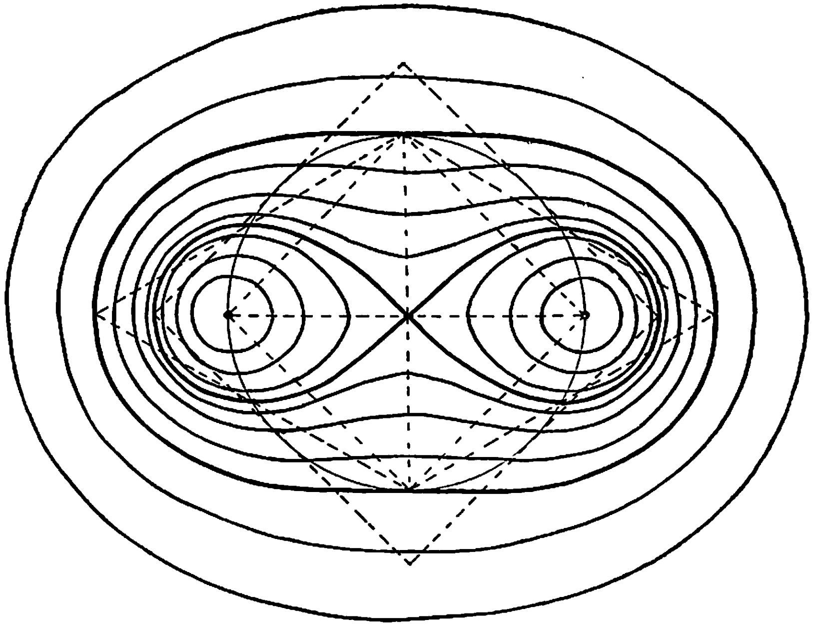


FIG. 8

From this figure we may obtain many interesting geometrical relationships, especially with regard to the flat oval and the Lemniscate. These are shown by the constructional circle and dotted lines. It is also interesting to notice that if we take the Lemniscate as being typical of these "product" curves we have in its centre part, where the two branches cut one another at right angles, the sign for multiplication (×).

In Fig. 9 we again have the same set of curves with the normals drawn from the two foci (the normals are lines which cut the curves at right angles).



The full lines (the normals) give a picture analogous to that which is obtained when one traces the "lines of force" of two like magnetic poles placed close together. (This may be done by placing a piece of paper over two bar magnets with their North (or South) poles in conjunction and sprinkling iron filings over the paper; on tapping the paper the filings will arrange themselves in definite lines, i.e. along the "lines of force.") Or if we consider two equal and similar electrical

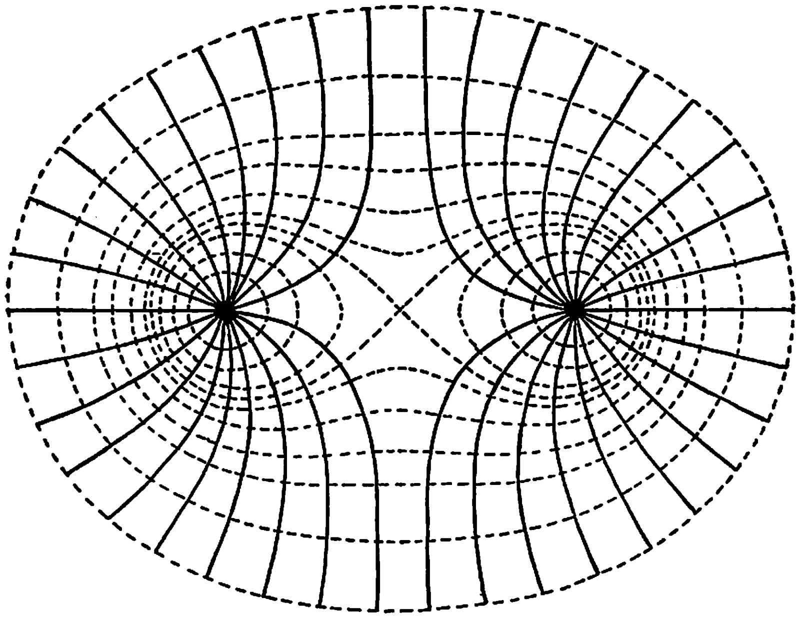


FIG. 9

charges at the foci, then the normals represent the lines of force due to these two charges and the dotted curves represent the lines of equal potential. (These curves of Cassini and their normals are mathematically not quite the same as the "electrical curves," but the analogy is very close indeed.) So we see here in the normals a picture of two opposing forces clashing and repelling one another. Or again in the pointed egg-shapes of the ovals we can see that they have only just separated themselves from the unity of the "mother" figure and are straining towards one another; while when they are pulled further apart they influence one another less and hence are more rounded.

While the children are drawing these figures they will feel and understand these relationships to the outer world and see that Geometry is not just something self-contained and cut off from the processes of Nature, but that it is living and working in all that goes on around them.

*Division.*

The drawings are here represented in one picture. The foci are fixed and the quotient varies. Thus, if we divide the focal

*Division.*

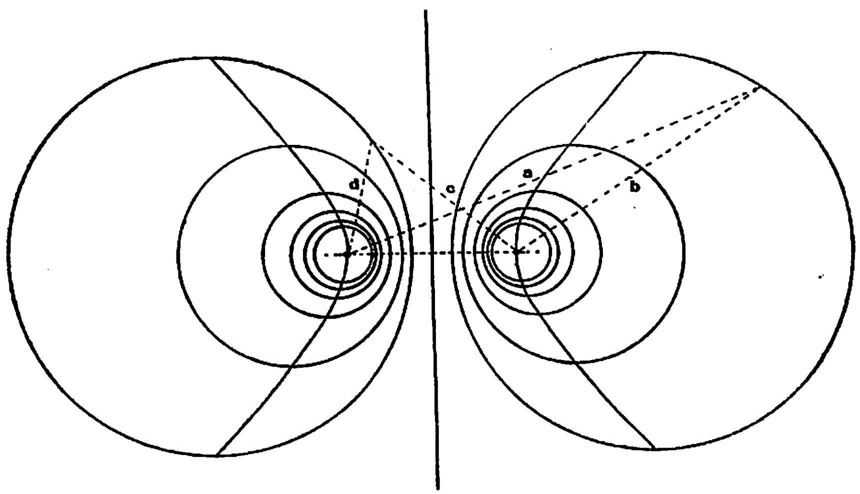



FIG. 10

distances of all points on the two large circles we obtain the constant quotient of $1\frac{1}{2}$ (for this drawing):

$$a \div b = c \div d \ (= 1\tfrac{1}{2}).$$

As the quotient gets larger, the circles become smaller and further apart and their centres approach the foci.

These circles, which are a picture of the process of division, are known as the circles of Apollo.

Again in division, the law of commutation is not valid (i.e. $a \div b$ does not equal $b \div a$) and so the functions of the two



foci are exchanged and the reality only lies in one of the pair of circles, the other one being the mirror-picture.

There is no change of form as in the case of the figures of multiplication. In the limit when the quotient is infinite, the circles become points at the foci. The other limit for the drawing is when the quotient is unity; then the circles become infinitely big and touch, i.e. the limit is a straight line which bisects at right angles the line joining the foci. It is interesting to notice that in passing from a quotient of $1\frac{1}{2}$ to a quotient fo 1—only a difference of $\frac{1}{2}$—the geometrical figure passes from quite small circles to infinite circles.

The rectangular Hyperbola (see Fig. 10) is obtained by joining by a smooth curve the highest and lowest points of the circles.

So the picture of division is not a circle but a pair of circles, in the same way that the picture of subtraction is not one branch of the Hyperbola but the two branches. If a pair of circles is filled in and the figure is turned so that the straigh line down the centre is horizontal, then we have the sign of division ( $\div$ ).

Again, these Apollonian circles may be connected with many phenomena in outer Nature and in man himself. These considerations, however, would lead us beyond the scope of the present article. They have been dealt with in detail by Ernst Bindel, of the Waldorf School, Stuttgart, in his recently published book *Das Rechnen im Lichte der Anthroposophie.*

The various examples taken from teaching practice which have been described and illustrated will serve to show what is meant by an imaginative, pictorial treatment of Geometry in education. Particularly in the last example one has been dealing with something fundamental; for these pictures of the four basic processes of calculation contain hidden within them, profound secrets of Nature and of Man. It has only been possible here to hint at these things. To penetrate to their hidden depths, a whole spiritual science of the Universe, Earth and Man is necessary.

In giving such drawings to the children one is not giving them something arbitrary, but something with a real, deep content. The connections with outer Nature which the teacher indicates are not fanciful but fundamental realities. So the children will gain their knowledge of Geometry on a sound and sure basis, and it will be something living and vital for them when, after the fourteenth year, they come to have a more intellectual understanding as the complement of what they have been experiencing imaginatively.

A. R. SHEEN.

(The drawings in Figs. 1 and 4 are reproduced by kind permission from *Geometrie in Bildern*, by Dr. Hermann von Baravalle, of the Waldorf School, Stuttgart.)

## *Alleluia for All Things*

OF all created things, of earth and sky,
Of God and Man, things lowly and things high,
We sing this day with thankful hearts and say,
Alleluia.

Of Light and Darkness and the colours seven
Stretching their rainbow bridge from earth to heaven
We sing, &c.

Of Sun and Moon, the lamps of Night and Day,
Stars and the Planets sounding on their way,
We sing, &c.

Of Times and Seasons, evening and fresh morn,
Of Birth and Death, green blade and golden corn
We sing, &c.

Of all that lives and moves, the Winds ablow,
Fire, and old Ocean's never-resting flow
We sing, &c.

Of Earth, and from earth's darkness springing free
The flowers outspread, the Heavenward reaching tree
We sing, &c.

* Written for the opening of the New School Hall.



Of creatures all, the eagle in his flight,
The patient ox, the lion that trusts his might,
We sing, &c.

Of Man, with hand outstretched for service high,
Courage at heart, truth in his steadfast eye
We sing, &c.

Of Angels and Archangels, Spirits clear,
Warders of Souls, and Watchers of the Year
We sing, &c.

Of God made man, and through man sacrificed,
Of Man through love made God, Adam made Christ,
We sing this day with thankful hearts and say
Alleluia

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