Historical Evidence for Energy Consumption Rebound in 30 US Sectors and a Toolkit for Rebound Analysts

Harry D. Saunders
Managing Director, Decision Processes Incorporated
2308 Saddleback Drive, Danville, CA 94506
e-mail: hsaunders@decisionprocessesinc.com

ABSTRACT

This article presents a detailed econometric analysis of historical energy consumption rebound magnitudes in the US economy by sector and in aggregate. The results appear to challenge the projections of the IPCC and call into question the potential for energy efficiency gains to provide a significant source of climate change remedies. Posted online alongside this article is a toolkit that allows any analyst to conduct a comparable analysis for any country, or sector, for which the data are available.

Introduction

The Intergovernmental Panel on Climate Change (IPCC) of the United Nations projects that by 2030 energy efficiency gains will provide a substantial part of the remedy for climate change by reducing global energy consumption approximately 30% below where it would otherwise be—nearly sufficient to offset projected economic growth-driven energy consumption increases.[1,2] This conclusion is supported by several analyses that project comparable energy reduction benefits arising from deploying visible "below cost" energy efficiency technologies.[3] If true, this means substantial carbon-reducing benefits are potentially available from energy efficiency gains that carry no cost (or even gains) to global economic welfare. This is a seductive concept, and one that shapes and informs much of current climate change policy.

This article analyzes this question in the context of the historical record for the US economy. It is already well known from theoretical considerations that so-called "rebound" effects can mitigate the reductions in energy consumption associated with energy efficiency gains. This article addresses this question empirically.

The results of the analysis are disturbing. If these results are even remotely correct, they would seem to represent a deeply problematic challenge to the projections and remedies foreseen by the IPCC, and indicate that countries proposing to meet emissions targets by relying significantly on energy efficiency gains stand the chance of falling far short.

[1] IPCC Fourth Assessment Report, Technical Summary Figures TS.3 and TS.10.
[2] Pielke, Wigley and Green (2008) argue that reductions due to energy efficiency gains projected by the IPCC are actually more like 80% owing to the assumed technology gains already embedded in the IPCC's baseline scenario.
[3] For example, McKinsey & Company, "Pathways to a Low-Carbon Economy" (2009). Lovins (1988,1990,1998,2005) has advanced similar claims for several decades.

Instead, the results strongly suggest that much greater onus must be placed on developing clean, cheap, and abundant energy supplies if those carbon emissions projections seen by the IPCC are to be realized.

Like the IPCC report, the widely-cited Stern report (2007) is seen by many as reflective of current best thinking on the question of future energy consumption trends. The Stern report, relying on projections made by the International Energy Agency (IEA), offered that the "technical potential for efficiency improvements to reduce emissions and costs is substantial," and cited the IEA finding that "energy efficiency has the potential to be the biggest single source of emissions savings in the energy sector." [4]

However, neither study takes any account of rebound effects, nor even mentions them. This analysis indicates that this may represent a significant oversight in both studies.

At minimum, the results show that demand-side remedies that rely on energy efficiency gains (especially while offering that such will come at little, or no, cost to economic welfare) should prudently be treated with substantial, if not deep, suspicion.

Background

While the scholarly treatment of rebound effects goes back a century and a half to Jevons (1865), it was Brookes (1979,1990a,1990b,1990c,1992,2000,2004) who brought Jevons' work to the attention of modern economists. Brookes and Khazzoom (1980) are generally credited with establishing the modern awareness of rebound phenomena. Since the time of their contributions, the number of researchers devoting attention to this issue has grown to the point that they are now too numerous to mention.

That said, the reader interested in the modern development of the theory and applications would be well served by referring to the seminal articles by Greening and Greene (1998), Greening et al. (2000) and the collections published by editor Schipper (2000), more recent work by Sorrell (2007) and Sorrell and Dimitropoulos (2007), and by editors Evans and Hunt (2009) and editors Herring and Sorrell (2009). A recent literature review is to be found in Jenkins et al. (2010). Saunders (1992) contains the earliest attempt to cast the rebound phenomenon in a formal theoretical framework. Later contributions to the theory can be found in Saunders (2000a,2000b), and in Wei (2007,2010), while Allan et al. (2006), Hanley et al. (2006, 2009), Barker et al. (2007a,2007b) Turner (2009) and Anson and Turner (2009) have more recently taken the field to whole new levels of sophistication. Saunders (2009) contains an undergraduate-level treatment of the theoretical foundations.

The literature is becoming vast, although curiously it currently largely originates in Europe.

The current article benefits greatly from the contributions of all these researchers, and many others.

[4] The specific IEA projections used by Stern are not specified in the Stern reference cited here, but more recent IEA projections (2009) claim that energy efficiency gains will contribute 40% to carbon emissions reductions by 2020 (see Figure 3, page 18).

Methodology, Briefly

In very basic terms, for each of 30 US sectors a four-factor (K, L, E, M) Translog unit cost function is measured econometrically, including measured technology gain parameters for each factor ($\lambda_K, \lambda_L, \lambda_E, \lambda_M$). The data set covers the period 1960-2005. Then, "backcasting" emulations are undertaken, using this measured cost function to depict producer choices, that explore what would have happened to energy consumption had energy efficiency gains ceased in 1980 (that is, had λ_E suddenly become zero). This depicts a situation of 100% rebound (that is, the energy consumption level that would have obtained had there been no energy efficiency gains beyond 1980). A second emulation, also using the same measured cost function and further described below, is used to depict a zero rebound condition. These two emulations plus the actual energy consumption allow a calculation of historical rebound over this period. The resulting rebound measurements are designated in this article "Rebound – Energy technology only."

A further set of emulations asks the same "what if" question, but under an assumption that no technology gains occurred *at all* after 1980 (that is, assuming $\lambda_K, \lambda_L, \lambda_E, \lambda_M$ suddenly became zero). The resulting rebound measurements are designated in this article "Rebound – all factors."

Results

The analysis econometrically measured historical rebound magnitudes in each of 30 individual US sectors, relying on the Jorgenson et al. data set covering the period 1960-2005.[5]

Figure 1 below shows the results aggregated across all sectors.

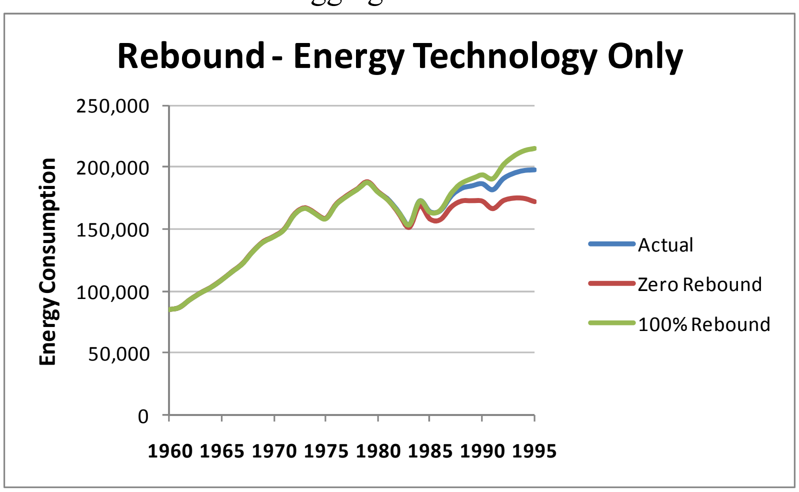

Figure 1. Historical Rebound in the US Economy

[5] Throughout this article, the phrase "Jorgenson et al." is used to designate both the data set (available at Dale Jorgenson's website http://dvn.iq.harvard.edu/dvn/dv/jorgenson;jsessionid=5f474ccc4bb102aec4ff946761c8) and the econometric approach developed by Jorgenson and his colleagues. The approach can be found in Jorgenson (2000) and Jorgenson, Ho and Stiroh (2005).

This figure, over three years in the making, shows energy consumption for three different scenarios: one, actual energy consumption; two, the energy consumption that would have occurred had measured energy-specific efficiency gains ceased in 1980 (labeled "100% rebound"); and three, the energy consumption that would have occurred were measured efficiency gains to have realized their full "engineering" potential (i.e., were an x% increase in energy efficiency to have resulted in an x% decrease in energy consumption below where it would have been without the efficiency gain), again starting in 1980 (labeled "zero rebound").

The labels describe three rebound conditions. In the parlance of rebound analysts, a rebound of 100% would be a situation where energy efficiency gains leave energy consumption unaltered compared to their absence. A zero rebound condition is one in which efficiency gains "take" on a one-for-one basis. (Technically, this is equivalent to assuming Leontief production technology, wherein an efficiency improvement in any one factor of production leaves economic output, output cost, and other factor uses unaltered[6]—and this is how it is modeled.) Rebound can then be quantified as the fractional "distance" actual energy consumption attains between the zero rebound condition and the 100% rebound condition.

By this analysis, the US economy exhibited substantial energy consumption rebound over this time period. The analysis shows rebound to have averaged 121% in the 1980-85 time frame (a condition known as "backfire"), 75% in the 1985-90 time frame, and 60% in the 1990-95 period. While this declining trend appears in the aggregate, as shown later the trend can be either declining or increasing in individual sectors.[7] These rebound trends have in part to do with the specific movements of factor prices over this time period.

Note that this analysis includes the productive part of the economy only—that is, those sectors of the economy that produce goods and services—and excludes end-use energy consumption by households and for personal transportation. But this brings to light a key point that is often a source of serious misunderstanding. Many policy makers (and not a few analysts), when contemplating the potential for reduced energy use, quite naturally reference their thoughts around those opportunities they see around them in their own personal realm. But this "direct" end-use energy consumption represents only a relatively small fraction of the energy they actually consume. Globally, some *two-thirds* of all energy is consumed "indirectly"— in the energy used to produce the goods and services consumed.[8] Your washing machine may be very efficient in its use of energy, but think of the metal body alone and the energy required to mine, smelt, stamp, coat, assemble and transport it to the dealer showroom where you bought the appliance. The energy "embedded" in your washing machine is substantial. The same is true for any product you purchase or service you consume.

Others (notably Greening and Greene, 1998, 2000) have evaluated measured rebound magnitudes for end-use consumption and have found them to be modest (in the order of 5%-

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[6] Saunders (2008).

[7] This declining trend refutes assertions based on theoretical considerations made by Saunders (2008) that long-term rebound must be greater than short-term rebound and supports assertions based on theoretical considerations made by Wei (2010) that they need not be.

[8] ExxonMobil (2009).

20%). But it is likely that rebound conditions are different as between "direct" and "indirect" energy consumption, so measuring rebound there is different from measuring it here. So this analysis, while it excludes end-use energy consumption by households and for personal transportation, addresses the far larger part of the energy economy where rebound phenomena are more difficult to discern.

All this strongly suggests that a simple-minded analysis that identifies a set of existing or prospective "below cost" efficiency technologies, assumes something about their deployment, and merely aggregates these into some quantity of energy assumed to be "saved" in the future is likely to be seriously misleading.

The story gets worse. The figure above considers only those technology improvements that are specific to energy inputs. But technology gains are not limited to those that improve energy efficiency. Instead, capital, labor and materials are generally prone to technology improvements that increase the effectiveness of their use. Theoretical considerations (e.g., Saunders, 1992) indicate that increasing the efficiency of these other factors is likely to increase energy demand. To ignore the possibility of these other technology gains in projections of energy consumption is at minimum problematic. Moreover, it is often the case that technologies aimed at improving energy efficiency also have the effect of increasing the efficiency of other factors of production. In fact, it is frequently argued that this is an ancillary benefit of new energy efficiency technologies.[9]

Figure 2 below shows the energy rebound effect of measured technology gains in all four factors of production (capital, labor, energy and materials):

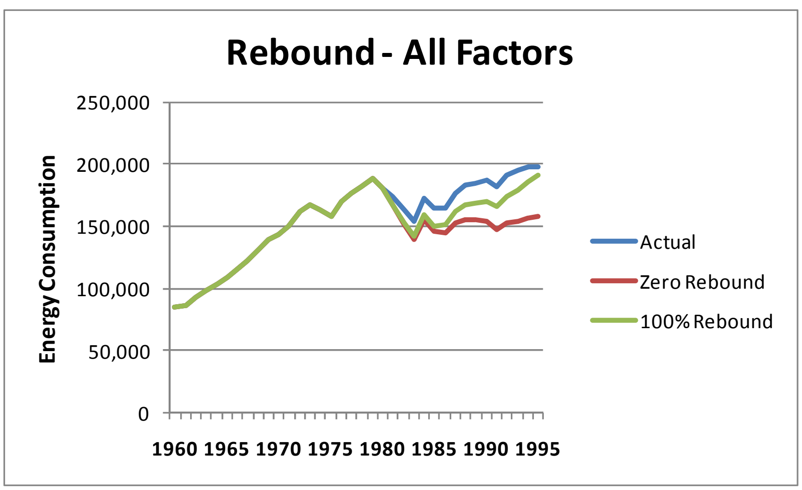

Figure 2. Historical Rebound in the US Economy due to All Technology Gains

This figure indicates that actual energy consumption was historically *above* what would have occurred were there no gains in technology for any factor of production. In other words, technology gains for all factors considered together created a condition of energy consumption "backfire"—technology improvements *increased* energy consumption rather than decreased it.

[9] See, for instance, Lovins (2005).

This strongly suggests that policy makers and analysts will be remiss in their duties if they fail to include consideration of technology gains for all factors in considering and making projections of future energy consumption trends.

Energy Intensity

Many analyses of future energy consumption rely on extrapolating trends in energy intensity, that is, trends in the ratio of energy use to economic output (often GDP is used as the measure of economic output).

A typical approach to estimating future energy consumption is to first make a projection of economic output and then to essentially multiplicatively apply some trend of energy intensity to deliver the projection of energy consumption, since $E \equiv Y\left(E/Y\right)$, where E is energy use and Y is (real) economic output. This is not dissimilar to the approach developed over 30 years ago by Lichtblau.[10] But it is a far too simplistic approach to match the significance of the task at hand – understanding remedies available for climate change.

Figure 3 below shows the energy intensity trends associated with the three rebound scenarios of Figure 2:

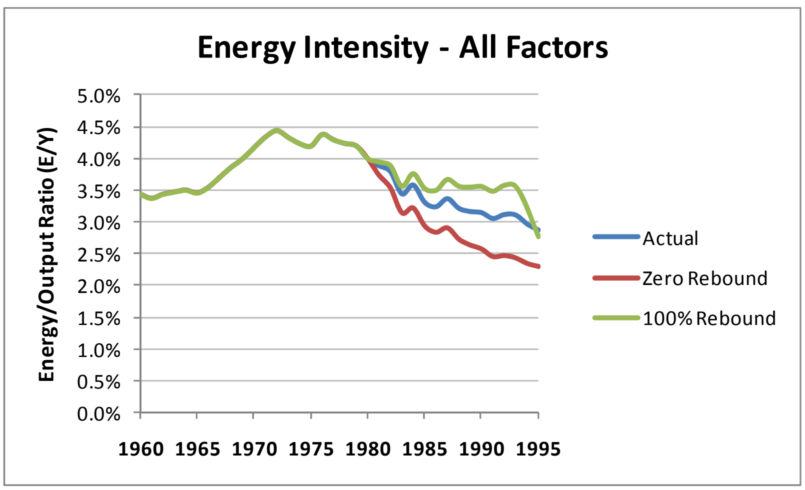

Figure 3. Energy Intensity Trends with different Rebound Scenarios

Energy intensity trends are determined by numerous drivers. Among these are relative movements in all factor prices, substitution possibilities among all factors, capital turnover, and technology trends for all factors. Further, different rebound scenarios are accompanied by different trajectories of economic output. A simple projection that does not account for all these considerations will not be a reliable projection.

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[10] John Lichtblau in the late 1970s and throughout the 1980s made numerous presentations to conferences of the International Association for Energy Economics that used such a methodology. While literature references are hard to come by, he is currently Honorary Chairman of Energy Policy Research Foundation, Inc. (http://eprinc.org) and may have copies of some of these presentations.

This point is forcefully brought home by observing that in Figure 3, the actual *intensity* trajectory lies *below* the intensity trajectory associated with 100% rebound, while in Figure 2, the actual energy *consumption* trajectory lies *above* that associated with 100% rebound (because output in the actual case is larger).

More generally, intensities for the other factors (capital, labor, materials) show declining trends in most sectors. But a declining intensity trend for any factor is not an indication that its associated technology improvement is factor saving; only that it is factor augmenting.

Importantly, this shows it is not possible to simply look at the historical trend of energy intensity and predict its path into the future. Even if one presumes to be able to predict with certainty future technology gains and future factor prices, one cannot look at an energy intensity trend and extract from it meaningful information about the historical or future role of energy efficiency gains.

Results – Detail

While rebound trends aggregated across sectors are surely of significance, there is much to be gained by looking at individual sectors.

Table 1 shows rebound results for each of the 30 sectors examined:

	Sector	Energy Use Share of Economy in 1980	Output Share of Economy in 1980	Energy-specific Rebound		All Factors Rebound		Energy-specific Rebound Components				All Factors Rebound Components			
								Short-term		Long-term		Short-term		Long-term	
				Short-term	Long-term	Short-term	Long-term	Substitution/ Intensity	Output	Substitution/ Intensity	Output	Substitution/ Intensity	Output	Substitution/ Intensity	Output
		%	%	%	%	%	%	%	%	%	%				
30	Electric Utilities	20%	3%	378%	120%	868%	169%	96%	4%	75%	25%	97%	3%	66%	34%
28	Transportation	16%	5%	105%	59%	889%	179%	97%	3%	87%	13%	97%	3%	73%	27%
34	Services	10%	16%	-16%	25%	164%	179%	102%	-2%	90%	10%	98%	2%	90%	10%
15	Chemicals	8%	7%	20%	53%	285%	147%	88%	12%	38%	62%	92%	8%	67%	33%
6	Construction	6%	10%	31%	58%	120%	135%	98%	2%	94%	6%	97%	3%	90%	10%
20	Primary Metal	6.0%	4%	185%	66%	1472%	172%	98%	2%	84%	16%	98%	2%	71%	29%
1	Agriculture	4.8%	0%	79%	39%	388%	381%	90%	10%	47%	53%	89%	11%	63%	37%
33	Financial Industries	3.8%	14%	39%	61%	618%	190%	98%	2%	95%	5%	97%	3%	86%	14%
35	Government Enterprises	3.6%	2.1%	93%	40%	955%	182%	98%	2%	87%	13%	98%	2%	75%	25%
7	Food & Kindred Products	2.7%	5.7%	26%	40%	393%	338%	99%	1%	98%	2%	98%	2%	94%	6%
13	Paper & Allied Products	2.7%	2.2%	65%	44%	472%	69%	96%	4%	80%	20%	97%	3%	73%	27%
19	Stone, Glass, Clay	2.3%	1.2%	101%	55%	636%	64%	97%	3%	82%	18%	97%	3%	63%	37%
22	Machinery, non-Electrical	1.6%	4.7%	11%	14%	503%	67%	92%	8%	71%	29%	94%	6%	17%	83%
21	Fabricated Metal	1.6%	2.9%	64%	40%	1193%	164%	99%	1%	96%	4%	99%	1%	89%	11%
23	Electrical Machinery	1.3%	3.1%	26%	41%	367%	81%	98%	2%	95%	5%	93%	7%	47%	53%
11	Lumber and Wood	1.1%	1.5%	36%	45%	594%	89%	98%	2%	89%	11%	97%	3%	73%	27%
17	Rubber & Miscellaneous Plastics	1.0%	1.3%	60%	37%	816%	133%	99%	1%	93%	7%	98%	2%	85%	15%
9	Textile Mill Products	1.0%	1.3%	37%	37%	403%	34%	96%	4%	89%	11%	94%	6%	-7%	107%
24	Motor Vehicles	0.8%	2.6%	25%	29%	772%	235%	99%	1%	97%	3%	97%	3%	80%	20%
5	Non-metallic mining	0.9%	0.2%	137%	54%	583%	76%	95%	5%	73%	27%	96%	4%	44%	56%
29	Communications	0.7%	2.7%	71%	60%	257%	104%	99%	1%	100%	0%	89%	11%	54%	46%
25	Transportation Equipment & Ordnance	0.6%	2.2%	22%	23%	383%	118%	99%	1%	96%	4%	97%	3%	86%	14%
14	Printing, Publishing & Allied	0.6%	1.9%	25%	25%	396%	56%	97%	3%	93%	7%	95%	5%	55%	45%
26	Instruments	0.6%	1.6%	33%	32%	222%	197%	86%	14%	51%	49%	90%	10%	72%	28%
10	Apparel	0.4%	1.2%	50%	52%	774%	255%	99%	1%	96%	4%	96%	4%	79%	21%
2	Metal Mining	0.3%	0.3%	131%	51%	613%	74%	95%	5%	73%	27%	96%	4%	46%	54%
12	Furniture & Fixtures	0.3%	0.7%	41%	19%	909%	230%	99%	1%	96%	4%	98%	2%	88%	12%
27	Misc. Manufacturing	0.3%	0.7%	32%	27%	468%	59%	98%	2%	95%	5%	97%	3%	62%	38%
18	Leather	0.1%	0.3%	35%	30%	434%	125%	99%	1%	97%	3%	97%	3%	84%	16%
8	Tobacco	0.06%	0.3%	38%	46%	136%	104%	91%	9%	70%	30%	87%	13%	70%	30%
	OVERALL *	100%	100%	126%	62%	649%	172%								

* Overall Rebounds are Energy Share-weighted

Table 1. Energy Rebound by Sector

In this table sectors are ordered in decreasing order of their 1980 energy consumption (sector numbers are those adopted by Jorgenson et al.). There is significant variation in rebound magnitudes as among sectors, but all sectors show substantial rebound for energy-specific technology gains and most show persistent backfire when technology gains for all factors are considered.[11] Sectors show a mixture of declining and increasing rebound over time ("short-term" refers to the period 1981-1990; "long-term" refers to the period 1991-2000). However, sectors with the largest consumption of energy have a dominant effect; thus, the trend for the aggregation across all sectors showing in Figure 1 is declining.

Columns to the right show the breakdown of rebounds as between the output component and the substitution/intensity component.[12] Note that these are proportional contributions that sum to 100%, to allow ready comparison across sectors. (Multiplying these by the rebound numbers delivers their contributions in absolute terms.) As is evident, the output component is the smaller component, contributing on average around 5% to rebound in the short term. This supports claims made by Schipper and Grubb (2000), Saunders (2000a), and Wei (2007). However, in the long term, the output component becomes more of a determinant of rebound, especially when technology gains for all factors are considered.

Time dynamics—decreasing and increasing rebound and output effects

The perhaps surprising variance among sectors of the time evolution of rebound and the increasing significance of the output component over time beg for an explanation.

Appendix B provides a detailed analysis of these time trends, but it is there shown that rebound time dynamics depends fundamentally on three determining elements: one, the inherent rebound propensity of the newest vintage of productive capacity, which in turn depends on the time evolution of its factor shares and substitution elasticities as they respond to technology gains and prices; two, the sectoral magnitude of technology gains; and three, the response of consumers to declining unit cost (price) of sectoral output following from the technology gains. As quantitatively illustrated there for several sectors, these determining elements behave differently in different sectors and the first element can even show oscillatory rebound behavior. At root, the differences lie in sectors' different measured cost/production functions.

The trend of increasing significance of the output component is more directly explained. Technology gains increase the productive capacity of each new vintage and this effect cumulates over time as vintages work their way through the sector and as output gains continue to materialize for each new vintage.

Sensitivity to Output Demand Elasticities

Rebound magnitudes are dependent on consumer response to declining prices of sectoral outputs. The 30-sector-wide aggregations showing in Figures 1 and 2 (and the results showing in Table 1) rely on an assumption that consumers exhibit Cobb-Douglas utility across sectoral outputs. As

[11] Note that small differences in absolute magnitudes of energy use can deliver large percentage-wise rebounds.
[12] See Saunders (2008) for definitions.

explained in a later section, this is the only way the aggregation can here be done while honoring general equilibrium principles. However, to pre-empt concerns that relaxing this assumption could reverse the conclusions, sensitivity analysis (of a fashion) can be invoked. The figure below illustrates:

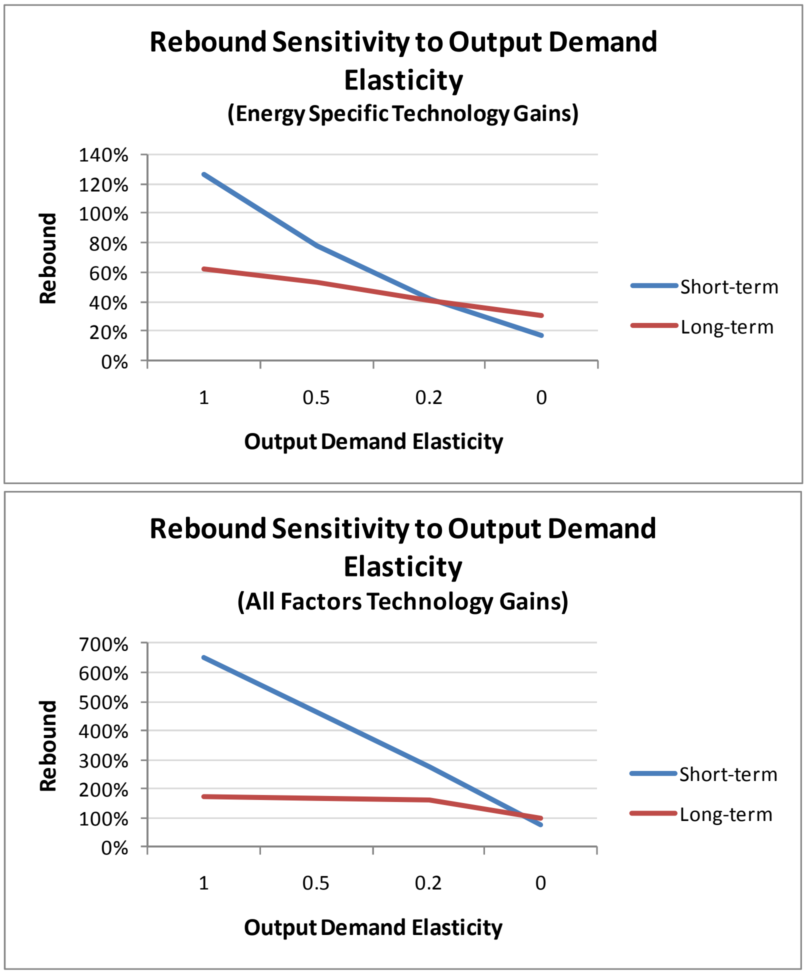

Figure 4. Rebound Sensitivity to Consumer Behavior

This figure shows how aggregate rebound magnitudes change as consumer elasticity of demand for output is reduced across all sectors. This is not a truly legitimate sensitivity analysis since only the results showing for an output demand elasticity of unity depict a genuine general equilibrium condition. In reality, reduced demand elasticities for various sectoral outputs would introduce complex movements of demand among sectors. Instead, this figure simply shows the effect of reducing the elasticity simultaneously for each sector and weighting the rebound results by sectoral energy consumption magnitudes for aggregation purposes.

The top panel shows that reduced elasticity of demand for output leads to reduced rebound. Interestingly, the aggregate short-term rebound falls below long-term rebound for elasticity sufficiently small. But it requires an extreme assumption of zero elasticity (that is, no consumer increase in demand for output as the price of output falls) to reduce aggregate rebound to the 20%-30% range—still of a magnitude not to be ignored. Few would argue an elasticity below 0.2 is remotely plausible, yielding about 40% rebound. Further, as shown in the bottom panel, the zero elasticity extreme still results in rebound of 100% when technology gains for all factors are considered (i.e., results in energy consumption unaltered by the technology gains).

We can discern from these considerations that the underlying causes of rebound can be described as twofold: one, an energy efficiency gain, by reducing the effective price of energy, allows firms to configure new productive capacity in a way that uses more of it profitably; and two, the concurrent reduction in the price of output causes greater consumption of that output, thus "dragging up" energy consumption, which is met by the efficiency-enabled increase in production capacity.

Strengths of the Methodology

While the results reported above may appear relatively straightforward, the methodology required to develop them is rather substantive. The methodology itself is summarized in a later section and in Appendix A. Here the strengths of the approach are outlined.

- The results rely on econometrically measured unit cost functions for each sector. No assumptions are made about parameter values such as substitution elasticities or technology trends. These are measured.
- The data used are those developed by Jorgenson et al. These data were developed with an astonishing degree of meticulousness and thoroughness, as can be appreciated by reference to Jorgenson, Ho and Stiroh (2005). There is arguably no better data set available anywhere.
- The functional form of the cost functions used is "rebound flexible."[13] The function is a four-factor (K, L, E, M) Translog cost function. Local concavity has been imposed according to the method developed by Ryan and Wales (2000). Many energy analysts instead use a CES (Solow) production function whose functional form is analytically convenient but problematic[14] and nearly all make assumptions about its parameter values. Sorrell (2008) has made a strong case that much previous work aimed at estimating rebound magnitudes is often flawed by arbitrary choice of production function and by arbitrary selection of substitution elasticity values, both of which drive rebound results. This approach avoids such arbitrariness.
- The approach differs conceptually from previous approaches that apply cost functions to sectors considered as a whole. Instead, the approach envisions that there is for each sector an underlying set of production possibilities available to firms contemplating new investment in capital assets (which possibilities can change over time with new technologies). The cost function visible to investors (dual to the underlying production function) thus applies only to new capital vintages. In traditional language, the model is

[13] See Saunders (2008) for the meaning of this term.
[14] See Saunders (2008).

a "putty-clay" model of production. This requires some fancy footwork to measure the incremental value shares associated with each new vintage of capital.

- The approach further differs conceptually from previous approaches by acknowledging that observed factor consumption *levels* and output *levels* (only these are visible in the data) are in general different from the *capacity in place* to use/produce them (not visible in the data), which is what production/cost functions measure. To deal with this fact, data on sector utilization rates[15] are used along with an array of different approaches to applying these utilization rates across vintages (vintages and their factor/output characteristics are individually tracked over time).
- The analysis is not bound to specific assumptions about capital formation but rather examines several possible theoretically plausible approaches for each sector, seeking the approach that offers the best explanatory power.
- Likewise, output capacity of the newest vintage is not tied to a single set of assumptions but rather the analysis examines several possible theoretically plausible approaches for each sector, again seeking the approach that offers the best explanatory power.
- The aggregation of factor uses and outputs across all sectors is done in a way consistent with general equilibrium theory. While as described in the following section this requires appealing to a "poor man's" general equilibrium model, it is the case that the aggregation honors the requirement that all factor markets and output markets clear at the specified (or calculated) factor and output prices, which is practically definitional of general equilibrium.

A further strength of the methodology is that the approach relies on extensive use of duality theory, which has a long and honored history in microeconomics. As described further in the Methodology section below, developing results expressed in "primal" terms (factor quantities and output quantities) requires back-and-forth transformations between the "primal" world and the "dual" world (factor prices and output prices). Duality theory provides the means to do this consistently and with theoretical integrity.

The case easily can be made that these methodological features represent a *minimum* set of considerations that must be accounted for if historical rebound is to be properly measured. The same can of course be said for methodologies for projecting future energy consumption trends. None of the models used by the IPCC for its energy consumption forecasts considers all, or even most, of these methodological features and the IPCC methodology essentially "force fits" the models to pre-determined scenarios. These models have remarkable abilities to connect multiple drivers to carbon emission projections, but no amount of detail can make up for an overly simplistic treatment of energy consumption that ignores or arbitrarily depicts rebound effects.

One final methodological point. The energy consumption trajectory shown as "Actual" in Figures 1 and 2 and used for developing the rebound measures in Table 1 is actually an

[15] Source: Federal Reserve utilization data available at http://www.federalreserve.gov/releases/g17/download.htm .

estimate of the actual. To provide a fair comparison among rebound trajectories, the same model was used to project actual energy consumption as was used to project the 100% rebound and zero rebound trajectories. In individual sectors, projected actual energy consumption deviates slightly from the true actual, sometimes somewhat above, sometimes somewhat below. However, in aggregate the estimated actual is virtually indistinguishable from the true actual, as shown in Figure 5.

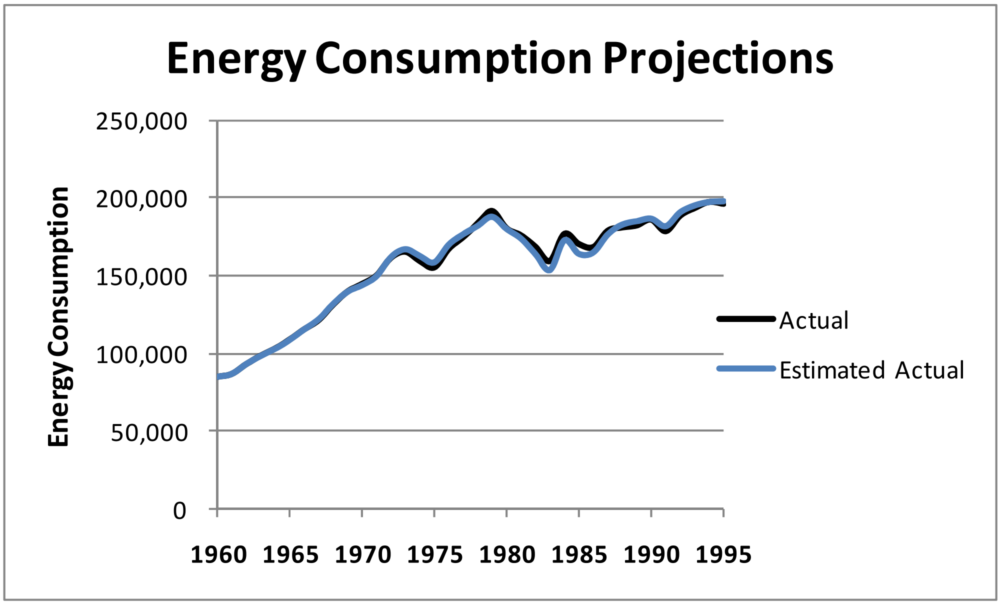

Figure 5. Actual vs. Estimated Actual Energy Consumption Trajectories

Given the multitudinous methodological elements involved in modeling these energy consumption trajectories, and the multiple excursions required back and forth between the primal and dual worlds, this is perhaps a testament to the reliability of the methodology, and perhaps a statement about the basic validity of the model's underlying assumptions of profit maximization and perfectly competitive firms.

Cautions and Limitations

The asserted strengths of the methodology do not mean it is without significant limitations. To enable econometrically-measured rebound estimates requires, at this point in methodology development, certain compromises. In addition to relying on the standard assumptions of microeconomics such as profit maximizing producers, perfect competition, and continuity, homogeneity and monotonicity of production/cost functions, the approach is limited in the following ways:

- The chosen functional form of the cost function is not the best possible candidate for an analysis of this type. While it is clearly superior to the CES (Solow) function and is highly popular among analysts, the Translog function is not as general as other forms developed by economists. As discussed elsewhere (Saunders, 2008), an ideal candidate would be the Gallant (Fourier) form, since this function does not presume membership in

- A limitation imposed by the use of Translog forms is that forcing concavity (a requirement generally called for by economists) presents some difficulties. It has been shown elsewhere (Saunders, 2008) that forcing global concavity on a Translog function necessarily leads to energy consumption backfire.[16] This limitation can be overcome by introducing constraints on the econometric measurement (discussed in the Methodology section) to force local concavity only. Ryan and Wales (2000) have provided such a methodology. However, this requires that the measured cost function be tested to see how well it behaves, concavity-wise. With the Ryan and Wales method, the measured cost function must be tested year-by-year post-measurement for each sector to evaluate its concavity over the relevant time horizon (and it depends on the reference year specified, since a different choice of reference year produces a different measured cost function). A rule-of-thumb metric is used to evaluate conformance with concavity.[17] But the results reported here include cases where not all sectors' cost functions honor concavity for every year.
- The factor technology gains measured in the analysis are assumed to be uniform over time, with smooth percentage technology gains for each factor each year. In reality, technology gains are more likely "lumpy," with new technologies appearing periodically in each sector. While the methodology can in principle accommodate year-by-year changes in technology, the added econometric parameters required would seriously degrade the statistical performance, and the statistical metrics generated using uniform technology parameters are already at the edge of respectability.
- For purposes of aggregating across sectors, a particularly simple assumption is made to permit conformance with general equilibrium theory. That is, consumer utilities are assumed to be of the Cobb-Douglas form. This creates a delightfully easy way to aggregate across sectors. With Cobb-Douglas utilities, when output prices and quantities change due to technology gains, demand for sectoral outputs can be treated independently and then summed together.[18] This provides a way to aggregate sectoral results that is consistent with general equilibrium conditions—but it must be skeptically considered a "poor man's" general equilibrium model.[19] A number of researchers (e.g., Turner, 2008,

[16] See Translog backfire theorem, Appendix F of Saunders (2008).
[17] Specifically, this metric is the sum of the positive eigenvalues of the core Hessian of the measured cost function over the relevant time horizon (modified as shown in Appendix A to comprehend the Ryan and Wales local constraints)—the larger this metric, the greater the departure from concavity.
[18] See, for example, Luenberger (1995), p. 132. Sector demand depends only on the sector-specific price of output.
[19] Robert Solow has called the Cobb-Douglas function the "Santa Claus" function. This application of it is yet another example of the "gifts" it provides analysts.

Wei, 2007,2010, Barker et al., 2007a,2007b) are hard at work to create more sophisticated, and realistic, general equilibrium models of rebound. So the aggregation presented here is best treated with considerable caution. That said, theoretical indications are that rebound is augmented to the extent substitution elasticities among factors is larger.[20] With the possibility of added factor substitution *among* sectors arising in a more robust general equilibrium model, this suggests that the Cobb-Douglas assumption may lead to an *understatement* of rebound magnitudes. Against this, because the Cobb-Douglas assumption implies unitary elasticity of demand for output, it could lead to an *overstatement* of rebound.

- In a similar vein, the analysis makes factor aggregation assumptions that are less than ideal. That is, while the cost of output, capital cost, factor demands and output levels are treated endogenously, labor and materials supply are treated as perfectly elastic. More specifically, prices applied in the rebound model for labor and materials are held fixed in nominal terms as between actual, 100% rebound, and zero rebound cases. This is not especially problematic for treatment of sectors individually since an individual sector is unlikely to have significant impact on factor supplies, especially those traded globally, and it is common to treat individual sectors as "price-takers." But aggregating across multiple sectors could be seen as more problematic. Since output levels will in reality generally be higher in the actual case than the 100% rebound case (due to factor efficiency gains), pressures on labor and materials supply will be greater, leading to higher labor and materials prices. However, this would have the effect of reducing the relative energy price, thus leading to a higher trajectory of energy consumption in the actual case, thereby *increasing* energy consumption rebound. It was felt that incorporating labor and materials supply functions in the analysis would run the risk of introducing an arbitrary element that could distort energy rebound in inscrutable ways. The methodology can accommodate depictions of labor and materials supply functions, but the risk of introducing what could be seen as arbitrary, or at least highly disputable, assumptions seemed high.[21] The bottom line: treating labor and materials supply as perfectly elastic likely *understates* actual energy consumption rebound.
- For energy, the situation is a little more complicated. A 100% rebound trajectory is one in which energy use is (generally) higher than in the actual case, at least for energy-specific technology gains. If a positive energy supply elasticity were introduced (undoubtedly the case in reality), the energy use trajectory in the 100% rebound case would accordingly be lower than if energy price remains unchanged, thus *increasing* rebound magnitudes. Offsetting this, a zero rebound condition would correspond with reduced energy consumption, thus reducing energy price and creating a higher trajectory of energy consumption in zero rebound conditions than without consideration of energy

[20] Saunders (1992,2008).

[21] To bound this problem, an attempt was made to include a feature in the Rebound Measurement Module that introduces perfectly *inelastic* supply for both labor and materials. Unfortunately, this is too severe, and invariably leads to instabilities in the projections.

price effects, and would thus *reduce* rebound magnitudes. This would have the effect of *reducing* the distance between the zero rebound and 100% rebound trajectories. While these effects may or may not be offsetting, for this analysis it was deemed prudent, as with labor and materials, to remain agnostic on the dynamics of energy supply (especially given the presence of OPEC as a non-competitive energy producer that sets global energy prices, making energy supply elasticity a complex concept), leaving for future analysts the task of comprehending the relative magnitudes of these offsetting effects. The toolkit methodology can accommodate an energy supply function, but the objective was to avoid hidden effects based on assumptions open to high controversy and dispute. Nonetheless, it is possible to argue that the rebound magnitudes reported in this article are distorted by this assumption (although whether the distortion understates or overstates rebound is a question that remains to be decided). That said, the assumptions of perfect (nominal) elasticity of labor, materials, and energy supply provide the means to aggregate across sectors in a way consistent with general equilibrium theory. Future analysts will no doubt have much to contribute here.

- The output of every sector is assumed to be directly consumed by end users. In reality, sectors use outputs from other sectors as inputs to produce their outputs. This "nesting" of outputs and inputs is entirely ignored in this analysis. While it is unclear what effect this shortcoming has on the magnitudes of measured rebounds, theoretical considerations suggest that it leads to an *understatement* of rebound. In particular, Lowe (2003) has shown that energy substitution elasticities become larger the more levels of nesting occur. As noted previously, theory indicates that larger energy substitution elasticities are associated with larger rebound magnitudes. Researchers such as Turner (2008,2009), Anson and Turner (2009), Allan et al. (2006), Hanley et al. (2006), Grepperud and Rassmussen (2004) and others are currently using models that better comprehend this "nesting" phenomenon. Lecca et al. (forthcoming) directly explore the consequences of different nesting schemes, helping fill a major gap in the field.
- The capital vintaging approach uses a "putty-clay" model. That is, while the newest vintage is deemed entirely flexible in choosing among the production possibilities represented by the cost/production function, once it is in place it is assumed to exhibit fixed factor and output capacity proportions, although factor and output magnitudes decline over time owing to depreciation. This is tantamount to considering older vintages as exhibiting Leontief technology behavior. This overlooks the potential for capital in place to be retrofitted in a way that changes its factor proportions. However, to the extent this reflects reality, it also suggests the potential for increased factor substitution across capital vintages. As noted above, increased factor substitution potential is theoretically associated with increased rebound, so the "putty-clay" approach likely *understates* rebound. This "putty-clay" approach is also undoubtedly more accurate than assuming new capital and capital in place are entirely fungible in a way governed by some sector-wide production function that includes both new and old vintages.

- Not all sectors represented in the Jorgenson et al. database are included in the analysis. Specifically, the analysis excludes coal mining and oil and gas extraction. The Jorgenson et al. data sets treat coal, oil, and gas as inputs to production in these sectors. While this makes perfect sense from a value-added perspective, these energy inputs are not actually consumed in these sectors, and so do not contribute to greenhouse gas emissions.[22] Similarly, the energy conversion sectors, petroleum and coal products (largely oil refining) do not consume the energy input, but rather transform it, so these sectors have been excluded as not consuming energy and creating associated emissions. The trade sector has also been excluded. The rationale for excluding it arises mostly out of the work of Allan et al. (2006) who have shown that this sector exhibits somewhat quirky behavior as regards rebound phenomena. That is, they have shown that energy efficiency gains in products that are exported can lead to greatly exaggerated local rebound effects. Government enterprises have been included even though it is doubtful this sector adheres to the assumption of profit maximization and perfect competition.
- The analysis considers only so-called "direct" rebound effects. That is, it includes both the output/income components of rebound and the substitution/intensity effects, both of which arise in the productive part of the economy. And it implicitly comprehends the phenomenon of consumers using savings from energy to purchase the output of other sectors ("indirect effects").[23] But so-called "macroeconomic" effects are excluded. The term "macroeconomic" is fraught with confusion and conflicting uses, but one clear example of a "macroeconomic" effect is that such as might arise when energy efficiency gains provide the basis for as yet unforeseen new energy-using applications, products, enterprises or even whole new industries. (This should probably be called the "Jevons effect."[24]) Accordingly, it is possible to argue that the results reported here thereby *underestimate* the economy-wide rebound arising from energy efficiency gains.
- There is judgment involved. The analysis relies on choosing among multiple theoretically plausible methods for depicting capital formation, new output-augmenting capacity, utilization rate profiles, and choice of reference year. For each sector, approximately 100 different method combinations are tested against the metrics of statistical performance, adherence to concavity, and minimum deviation of forecast projections of factor uses and output from actuals. That said, a remarkable thing is that method combinations that deliver good performance against one metric (e.g., concavity) also tend to deliver good performance against all metrics. Further, where performance is

[22] Energy is actually consumed in these sectors, and emissions produced, but it is not possible to separate out from the data set the portion of energy inputs actually consumed.
[23] Although with the Cobb-Douglas utility specification on consumer behavior assumed here, consumption is not reallocated among sectors.
[24] A good example of the "Jevons effect": Tsao et al. (2010) have shown that new applications in efficient lighting have, since the 1700s, offset the energy efficiency gains from new lighting technologies almost exactly, leaving energy intensity of lighting unchanged over hundreds of years and independent of "luminous efficacy." New lighting applications have continually arisen that offset energy consumption reductions due to energy efficiency gains, for more than 300 years, across 3 continents and across 6 technologies.

good for more than one method combination, measured rebound magnitudes are relatively stable and do not differ much one from the other. Accordingly, while choosing among method combinations involves judgment, it is unlikely that other researchers using this methodology would choose significantly different method combinations or report significantly different rebound magnitudes. Moreover, where more than one methodology provides reasonable explanatory power and plausible statistics, the methodology resulting in *lower* rebound has been used. Thus, the results may for this reason *understate* rebound magnitudes.

More generally, extreme effort has been made to avoid "cheating" whenever judgment is brought into play.[25]

- No consideration is given to producers employing "rational expectations" in their decision making. Rather producers are assumed to choose production technologies based on factor prices prevailing in the year the investment is made. While this is consistent with most models of energy consumption, it is a limitation.
- The analysis excludes consideration of what would have happened were carbon taxes or additional energy use regulations invoked during this time period. These have clear implications for forecasting future energy consumption trends and rebound effects. However, it is to be noted that such government interventions have the certain effect of reducing economic welfare (at least welfare narrowly construed to exclude externalities). Instead, this analysis indicates the effects on energy consumption of technology gains that do not come at a cost to economic activity. This seems the most honest way to evaluate pure rebound effects.
- Government monetary and fiscal policy is held fixed across the three rebound scenarios. In reality, for instance, had the 100% rebound case actually obtained, government may have invoked monetary or fiscal stimulus to offset the lower output trajectory associated with this case. Again, however, ignoring this seems the most honest way to provide an "apples-to-apples" comparison that isolates rebound effects.
- A final, important, caution: these results should **not** be taken as an argument against deploying new energy efficiency technologies. Such technologies increase economic welfare (narrowly construed to exclude externalities). It is just that they may not deliver the reductions in energy consumption presumed by many.

This impressive list of limitations should not be dismissed out of hand by rebound analysts as minor or irrelevant. Rather, the intent in delineating them is that practitioners need to find ways to overcome them if energy consumption rebound is to be properly understood.

[25] Robert Solow in the late 1970s gave a presentation at Stanford University called something like, "How to lie with econometrics" in which he showed the many ways in which it is possible for econometricians to fool themselves—or others. This had a profound and lasting effect on a young researcher.

Nonetheless, it is hoped that this analysis is seen as a serious, highly methodical attempt to measure historical rebound and to provide a practical toolkit to rebound practitioners that adheres to exacting theoretical standards.

Methodology

The methodology is best described by reference to the toolkit posted online with this article. This toolkit is highly automated and user-friendly and can be used by any analyst having access to data comparable to the Jorgenson et al. data at a sectoral level. The modules are replete with interactive graphics that allow the user to observe various performance measures and to toggle among different methods to see their effects. A User Guide accompanies the toolkit; media demos are available that provide the quickest way to understand its functioning. The toolkit further provides an audit trail for anyone desiring to replicate the results reported in this article.

The toolkit contains five interlinked modules, depicted conceptually below:

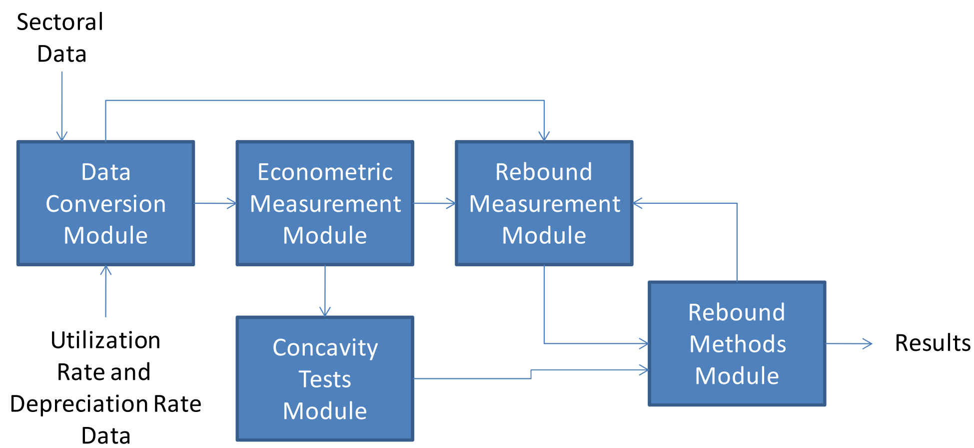

Figure 6. The Rebound Measurement Toolkit

Data Conversion Module

The Data Conversion Module (which in the toolkit provided contains the Jorgenson et al. data set[26]) takes the raw data and converts it to incremental value shares (for K, L, E and M) associated with each new vintage. As explained further in the Appendices, this involves keeping track of the vintages' factor and output *capacities* over time. Sector utilization rates must be applied for each year to these capacities to achieve conformance with observed annual sector-wide factor *consumption levels* and output *production levels* consistent with the capacities. When the incremental factor capacities are established that match sector-wide consumption levels, factor prices from the data are applied to these quantities to obtain incremental factor value shares. The incremental shares are input to the Econometric Measurement Module.

[26] All that is required for application to other economies/sectors is to insert the appropriate data in the appropriate columns/rows of the data worksheet.

The Data Conversion Module can be operated with different methods. For the application of utilization rates, three methods are available:

- *Uniform utilization*, wherein sector-wide utilization rates (from FRB data, mentioned previously) are applied equally across vintages.
- *Permanent shutdown*: Utilization rates for each vintage are corrected to exclude vintages with negative cash flow, which are assumed to be permanently removed from service.
- *Temporary shutdown*: Utilization rates as above, but with vintages allowed to return to service if they go cash flow positive.

The latter two seem on the surface the most plausible methods when one observes the operating philosophy implemented by most industrial firms, but a surprising result is that uniform utilization most often delivers the best econometric results and best conformance of factor and output forecast projections with actuals. The methods chosen in this article for various sectors are summarized in Appendix C.

The use of utilization rates requires an adjustment to be made to the capital prices from the data set (see Appendix A). But the module allows this to be toggled on and off. Using this adjustment almost always provides the best performance, although there are some sectors where the FRB does not report utilization rates and toggling this adjustment off produces better performance, probably because the *industry*-wide average utilizations applied in these cases is sometimes inaccurate.

The module allows the user to choose different reference years to scale the data. This is necessary for applying the Ryan and Wales procedure described later.

Finally, there are four choices for emulating capital formation.

- *Accounting calculation*: New capital is calculated as the difference between observed capital-in-place less last period's capital-in-place times the measured depreciation rate.
- *Revenue reinvestment*: Revenue realized by the sector is the product of output price and output, cY. Revenue is converted to capital quantity terms by dividing this product by the price of capital. Then, a revenue reinvestment rate is measured econometrically and multiplied by this quantity.
- *Cash flow reinvestment*: Sector cash flow is calculated as revenue less payments to all factors except capital. An econometrically-measured cash flow reinvestment rate is applied to this quantity to generate new capital.
- *Shephard's Lemma*: New capital for the period is calculated, from Shephard's Lemma, as the product of output price, real incremental output capacity and the previous period's incremental capital value share, divided by the price of capital.

Again, while the cash flow reinvestment rate method might *a priori* seem the most likely match to observed firm behavior, it rarely delivers the best results. The Shephard's Lemma method is by far the most common in delivering good performance. The methods chosen in this article for various sectors are summarized in Appendix C.

All these methods associated with the Data Conversion Module are described in more detail in Appendix A (and the User Guide).

Econometric Measurement Module

This module takes output from the Data Conversion Module and applies a Jorgenson et al.-style econometric approach (described in a later section) to measure the parameters of each cost function, with application of Ryan and Wales' approach to force (local) concavity by introducing added constraints on the cost function (see Appendix A for specifics).

The econometric analysis is a multi-stage procedure, with the final stage being an application of the Full Information Maximum Likelihood method invoking a Gaussian convergence search routine.

The application used is TSP®. The TSP code is provided in the Toolkit posted online. While it required considerable effort to create, the code is now surprisingly robust across the significantly different data sets of the different sectors. Over more than a thousand different runs, failure to converge was seen in perhaps three cases. This suggests it should be readily usable by analysts wishing to apply it to data available for different economies.

Concavity Tests Module

With the Ryan and Wales procedure, the cost function measured by the Econometric Measurement Module must be tested year-by-year for concavity performance. Because the procedure forces concavity only locally (in a specified reference year), and because the modified Hessian of the cost function is thereby dependent on value shares, which change year-by-year, this module invokes a procedure that calculates the largest-in-magnitude eigenvalue of the modified Hessian in each year.

This module automates this process. It picks up the cost function parameters and sectoral data from other modules and allows users to quickly conduct the tests with the click of a button.

In any one year, if the largest eigenvalue of the modified Hessian is negative, the function is strictly concave for that year; if the largest eigenvalue is zero, the function is quasi-concave; if the largest eigenvalue is positive, the function is convex. Judicious selection of reference year improves concavity performance.

Fortunately, for the results reported here, the procedure delivers concavity metrics indicating the measured cost functions are at least quasi-concave for most sectors, and moreover are not infrequently strictly concave over the domain of rebound measurement. This accords with the claims for the procedure advanced by Ryan and Wales (2000).

Rebound Measurement Module

This module takes the measured cost function (and measured factor technology gain parameters) and creates a "backcast" of energy consumption under three scenarios: one (actual), which applies the measurements to the sectoral data including all the technology parameters; a second (100% rebound), which applies the measurements with either the energy technology parameter or all technology parameters set to zero; and a third (zero rebound), which simply introduces the energy technology parameter directly to the newest vintage in Leontief fashion—reducing energy factor capacity each year by the annual technology gain while leaving the other factor capacities/output capacity at levels reflecting the 100% rebound case, and allowing these new vintages to work their way through the system over time.

As with the Data Conversion Module, this module allows application of several different methods, including the utilization rate and capital formation methods described above. In addition, this module contemplates different methods for emulating the magnitude of the output capacity of the newest vintage. Output capacity will differ as between actual, 100% rebound and zero rebound cases, since different technology gains apply in each of these cases. There are two basic methods. The second method has three available sub-methods:

- *Accounting method*: In this method, the incremental factor capacities in each year are calculated according to Shephard's Lemma and the incremental output capacity is calculated from the accounting identity:

  $$\Delta Y^t = \left(p_K^t \Delta K^t + p_L^t \Delta L^t + p_E^t \Delta E^t + p_M^t \Delta M^t \right) \Big/ c^t$$

- *Ohti's Theorem method*: This method comes in three forms, all based on an extension of Ohti's Theorem
  - Delta X form (prices)
  - Delta X form (shares)
  - Price elasticity form

  These three forms are rather involved and are explained in Appendix A.

This module also determines the effect of the consumer demand response. The module endogenously determines the unit cost by year, which will differ in each of the three rebound scenarios. It also endogenously determines the price of capital, which also will differ since incremental capital (investment) differs in each scenario (and which feeds into the unit cost calculation). The calculated unit cost permits introduction of consumer utilities to adjust consumer demand for the output from the sector, based on this calculated output price (unit cost).

With consumer budgets assumed unchanged,[27] an assumption of Cobb-Douglas consumer utility means the elasticity of demand for the sector's output with respect to its price is negative unity. By calculating the difference in unit cost for the 100% rebound case relative to the actual case, the relative change in demand for that sector's output can be calculated by a simple elasticity equation. Since the 100% rebound case will have a higher unit cost than the actual case (because the 100% rebound case assumes zero technology improvement) its output will be lower due to depressed consumer demand. The zero rebound case does not require such an adjustment since with the Leontief technology assumed there, the unit cost of output does not differ from the 100% rebound case given the technology gain

As noted previously, this use of Cobb-Douglas utility also enables sectoral outputs to be treated independently when aggregating them.

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[27] If one allows consumer budgets to change as the result of different rebound (technology) scenarios, the effect will be to lower both the 100% rebound trajectory and the zero rebound trajectory, thus increasing rebound magnitudes. For both scenarios the unit cost of output is higher than in the actual case.

Rebound Methods Module

This module manages the comparison of the multiple methods for measuring rebound. Once different cost function candidates have been econometrically measured, cost function parameters are entered into this module.[28] Then different rebound measurement methods are applied to each measured cost function. Each cost function is invoked with the click of a button, and the Rebound Measurement Module calculates rebound and other measures such as forecast deviation for each rebound method automatically.

For each sector analyzed here, about 100 different econometric/rebound method combinations were evaluated to explore the methods space.

Comparison to Jorgenson et al. Methodology

The econometric measurement procedure adopted here relies very heavily on the methods created by Jorgenson et al. (and would not have been possible without these).[29] But it differs in three significant ways.

One, to obtain cost function parameters, the procedure used here measures the *incremental* value shares associated with the newest vintage of capital only, as discussed above.[30]

Two, the procedure takes account of differences between the factor/output characteristics of *capacity in place* and *actual* factor uses/*actual* output as revealed in the data. Sector utilization rates are invoked to calculate the implied factor/output characteristics of the new vintage's capacity, as discussed above.

The third difference has to do with the way technology gains are characterized. In the standard Jorgenson et al. approach, technology parameters appear in terms appended to the core Hessian of the Translog. The resulting cost function looks like the following:

$$\ln c = a_0 + a_t t + \mathbf{a} \ln \mathbf{p}^T + \frac{1}{2} \ln \mathbf{p} \cdot \mathbf{B}_{pp} \cdot \ln \mathbf{p}^T + \ln \mathbf{p} \cdot \boldsymbol{\beta}_{pt} t + \frac{1}{2} \beta_{tt} t^2 \qquad (1)$$

where $\mathbf{a}$ is a vector of value shares (a_K, a_L, a_E, a_M), $\mathbf{p}$ is a vector of prices (p_K, p_L, p_E, p_M), $\boldsymbol{\beta}_{pt}$ is a vector of factor-related technology changes $(\beta_K, \beta_L, \beta_E, \beta_M)$, β_{tt} is a technology parameter reflecting secular drift in the cost function, and $\mathbf{B}_{pp}$ is the core Hessian matrix.

The approach used here is somewhat different. It considers first a Translog function in the absence of technology gains:

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[28] This is the one manual step: the TSP outputs must be copied and pasted into this module.

[29] A seminal article outlining this methodology is Jorgenson and Fraumeni (1981). A key reference is Jorgenson (2000).

[30] Since investors in new production capacity "make the calculation" based on prevailing output price (unit cost), and all vintages must accept this price (there being no flexibility to adjust factor proportions in old vintages), the unit cost function obtained from this method must reflect the sector-wide price of output. This result is confirmed by the close match between the measured cost function trajectory and the actual trajectory in each sector.

$$\ln c = \ln c_0 + \sum_{i=1}^{4} a_i \ln p_i + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} b_{ij} \ln p_i \ln p_j \tag{2}$$

where the indices correspond to the four factors. Then, it introduces factor-augmenting technology gains:

$$\ln c = \ln c_0 + \sum_{i=1}^{4} a_i \ln\left(\frac{p_i}{\tau_i}\right) + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} b_{ij} \ln\left(\frac{p_i}{\tau_i}\right)\ln\left(\frac{p_j}{\tau_j}\right) \tag{3}$$

(This is mathematically dual to introducing factor-augmenting technology terms to the production function, as in $Y = f(\tau_K K, \tau_L L, \tau_E E, \tau_M M, \cdots)$, for any CRS function.[31]) the τ_i are assumed to be of the form $\tau_i(t) = \tau_0 e^{\lambda_i t}$, indicating constant-over-time, smooth technology gains. With the introduction of the vector $\boldsymbol{\lambda} = [\lambda_K, \lambda_L, \lambda_E, \lambda_M]$, the equivalent to equation (1) is written:

$$\ln c = \ln c_0 - \mathbf{a}\boldsymbol{\lambda}^T t + \mathbf{a}\ln\mathbf{p}^T + \frac{1}{2}\ln\mathbf{p}\cdot\mathbf{B}\cdot\ln\mathbf{p}^T - \boldsymbol{\lambda}\cdot\mathbf{B}\cdot\ln\mathbf{p}^T t + \frac{1}{2}\boldsymbol{\lambda}\cdot\mathbf{B}\cdot\boldsymbol{\lambda}^T t^2 \tag{4}$$

where **B** is the core Hessian matrix. Note that there is a one-to-one correspondence between the terms of (1) and of (4) and specifically between those terms that relate to technology gains:

Jorgenson et al. form	Factor-augmenting form
$a_t t$	$-\mathbf{a}\boldsymbol{\lambda}^T t$
$\ln\mathbf{p}\cdot\boldsymbol{\beta}_{pt} t$	$-\boldsymbol{\lambda}\cdot\mathbf{B}\cdot\ln\mathbf{p}^T t$
$\frac{1}{2}\beta_{tt} t^2$	$\frac{1}{2}\boldsymbol{\lambda}\cdot\mathbf{B}\cdot\boldsymbol{\lambda}^T t^2$

The Jorgenson et al. form contains five technology parameters; the factor-augmenting form contains four. As seen above, the factor-augmenting form constructs the equivalent of the β_{tt} term from the four individual factor-augmenting technology parameters and the Hessian matrix. This raises a question as to the meaning of the extra β_{tt} parameter: it cannot represent neutral technology gain, since for any CRS function, neutral technology gain can be equivalently subsumed within the individual factor-augmenting technology gains; factor-augmenting technology gains create a term like this without appending an added parameter. This term appears to emulate something, but it is not clear what. Jorgenson et al. refer to it as "a constant deceleration of technical change."[32]

Whatever the theoretical difference represented in these two approaches, the factor-augmenting approach allows, as seen from the table above, calculation of what are here called "Jorgenson-equivalent technology parameters" using the measured factor-augmenting technology parameters. The two parameter sets can then be compared:

[31] See Saunders (2005), Supporting Proofs, Theorem 4.
[32] Jorgenson (2000), p. 14.

	Sector	Measured Factor-augmenting Technology Parameters				Measured Jorgenson-equivalent Technology Parameters				
		λ_K	λ_L	λ_E	λ_M	β_{KT}	β_{LT}	β_{ET}	β_{MT}	β_{TT}
		%/year	%/year	%/year	%/year					
30	Electric Utilities	2.28%	3.74%	2.34%	1.00%	-1.166E-03	1.612E-03	4.644E-04	-9.09E-04	3.558E-05
28	Transportation	2.26%	1.98%	1.95%	1.77%	-1.241E-04	-6.581E-05	7.18E-05	1.18E-04	-6.212E-07
34	Services	-0.52%	1.23%	1.65%	-1.20%	1.402E-03	1.512E-03	5.78E-05	-2.971E-03	4.779E-05
15	Chemicals	2.61%	5.19%	3.30%	0.41%	-4.345E-04	3.797E-03	4.14E-05	-3.40E-03	1.729E-04
6	Construction	2.95%	2.36%	2.01%	1.84%	-3.565E-04	-7.875E-04	2.266E-05	1.12E-03	-8.046E-06
20	Primary Metal	2.46%	3.30%	2.90%	0.53%	6.263E-03	4.949E-03	4.423E-04	-1.17E-02	2.685E-04
1	Agriculture	1.82%	0.03%	1.63%	0.15%	-2.401E-04	3.541E-04	-1.46E-04	3.195E-05	-6.600E-06
33	Financial Industries	5.32%	9.42%	11.73%	1.49%	2.397E-03	9.962E-03	1.340E-03	-1.37E-02	1.019E-03
35	Government Enterprises	2.25%	1.82%	1.79%	1.65%	-4.614E-04	1.047E-04	3.719E-05	3.195E-04	-2.535E-06
7	Food & Kindred Products	-0.56%	0.61%	0.71%	0.35%	3.308E-04	1.252E-04	-3.38E-06	-4.53E-04	-2.674E-06
13	Paper & Allied Products	1.32%	4.81%	2.53%	-0.34%	4.070E-03	3.017E-03	6.570E-05	-7.15E-03	2.247E-04
19	Stone, Glass, Clay	3.20%	1.64%	2.31%	1.64%	-7.233E-04	4.950E-04	-2.9886E-04	5.271E-04	-1.327E-05
22	Machinery, non-Electrical	4.70%	4.38%	4.17%	4.61%	-1.084E-04	5.104E-04	4.223E-05	-4.443E-04	-1.467E-06
21	Fabricated Metal	0.75%	0.67%	0.81%	0.66%	-6.637E-06	2.681E-05	-2.833E-05	8.153E-06	-4.599E-08
23	Electrical Machinery	6.41%	7.91%	4.08%	-2.02%	-1.545E-03	8.553E-03	7.054E-05	-7.078E-03	7.235E-04
11	Lumber and Wood	0.40%	0.94%	1.04%	1.02%	1.778E-03	2.841E-03	4.062E-06	-4.62E-03	1.791E-04
17	Rubber & Miscellaneous Plastics	7.97%	5.78%	3.14%	0.95%	2.052E-03	5.103E-03	-6.134E-05	-7.094E-03	3.896E-04
9	Textile Mill Products	2.88%	1.93%	1.63%	1.52%	-5.129E-04	-4.848E-04	2.526E-05	9.724E-04	-8.914E-06
24	Motor Vehicles	3.21%	5.33%	1.14%	0.17%	3.812E-03	2.900E-03	-2.352E-05	-6.689E-03	2.652E-04
5	Non-metallic mining	2.39%	1.07%	1.28%	0.99%	-2.168E-04	3.404E-05	-2.332E-05	2.061E-04	-3.078E-06
29	Communications	3.31%	4.28%	2.99%	0.52%	2.041E-03	5.441E-03	1.700E-05	-7.500E-03	2.619E-04
25	Transportation Equipment & Ordnance	5.06%	4.01%	10.64%	2.45%	-1.157E-03	1.103E-03	1.630E-04	-1.085E-04	3.834E-07
14	Printing, Publishing & Allied	4.11%	0.34%	1.36%	0.79%	-3.369E-04	7.423E-04	-3.838E-05	-3.670E-04	-1.475E-05
26	Instruments	2.84%	2.09%	2.54%	1.72%	-3.546E-04	-2.742E-04	-3.912E-05	6.680E-04	-5.344E-06
10	Apparel	0.15%	6.11%	12.85%	-5.61%	6.952E-03	8.758E-03	5.557E-04	-1.627E-02	1.530E-03
2	Metal Mining	29.59%	3.12%	2.36%	2.28%	-2.347E-04	-1.324E-04	6.852E-04	-3.181E-04	-6.467E-05
12	Furniture & Fixtures	4.44%	6.49%	3.44%	-0.92%	3.248E-03	6.854E-03	3.137E-05	-1.013E-02	6.840E-04
27	Misc. Manufacturing	2.06%	0.94%	0.77%	0.78%	-4.752E-04	6.457E-05	-1.551E-05	4.261E-04	-5.946E-06
18	Leather	2.56%	4.82%	1.76%	-0.64%	5.588E-04	2.425E-03	-1.115E-05	-2.973E-03	1.501E-04
8	Tobacco	3.21%	8.28%	16.56%	0.02%	7.930E-03	3.850E-03	2.130E-04	-1.199E-02	6.061E-04

Table 2. Measured Technology Parameters

Note that some sectors show negative factor-augmenting gains for some factors. This is not surprising, since it is very likely that in reality new technologies come in "bundled-together" forms that sometimes augment some factors at the expense of others.

The econometric measurement uses value share equations, derived from the unit cost equation by undertaking the operation $s_i = \dfrac{\partial \ln c}{\partial \ln p_i}$. For the factor-augmenting approach, this delivers

$$s_i = a_i + \mathbf{B}_i \ln \mathbf{p}^T - \mathbf{B}_i \boldsymbol{\lambda}^T t \tag{5}$$

where $\mathbf{B}_i$ is the row of $\mathbf{B}$ corresponding to the factor i. In the implementation reported here, it is actually *incremental* value shares (associated with the newest vintage) that are measured. The technology gains are treated as applying to the newest vintage only; older vintages have already chosen their technology. Introducing incremental notation, the equivalent of equation (5) can be re-arranged in a way that promotes easier interpretation:

$$s_{\Delta i} = a_i + \mathbf{B}_i \left(\ln \mathbf{p}^T - \boldsymbol{\lambda}^T t \right) \tag{6}$$

This formulation shows that share can be thought of as being determined by the combination of a price elasticity-based operator and an "effective price" operator, where by "effective price" is meant the price per unit of energy services as it would appear to the producer given the technology gain.[33] More specifically, this term is the change in effective price with time. The basic price-driven interactions to share (from the core Hessian) mediated by $\mathbf{B}_i$ remain unaffected by technology. Instead, one can think of the prices as themselves being modified by the technology gains, creating technology-modified share dynamics secularly trending over time.

This device also allows the unit cost equation to be cast in a more transparent form:

$$\ln c = \ln c_0 + \mathbf{a}\left(\ln \mathbf{p}^T - \boldsymbol{\lambda}^T t \right) + \frac{1}{2}\left(\ln \mathbf{p} - \boldsymbol{\lambda} t \right) \cdot \mathbf{B} \cdot \left(\ln \mathbf{p} - \boldsymbol{\lambda} t \right)^T \tag{7}$$

As with shares, the elasticities of the cost function's magnitude with respect to price and technology changes come from what can be thought of as "effective" price interactions.

With homogeneity imposed, only three of the share equations in a four-factor formulation are independent, requiring the addition of a fourth equation. The fourth equation is

$$\frac{d \ln c}{dt} = \mathbf{a} \cdot \left(\mathbf{dlnp}^T - \boldsymbol{\lambda} \right) + \frac{1}{2}\left(\ln \mathbf{p} - \boldsymbol{\lambda} t \right) \cdot \mathbf{B} \cdot \mathbf{dlnp}^T - \frac{1}{2}\left(\ln \mathbf{p} - \boldsymbol{\lambda} t \right) \cdot \mathbf{B} \cdot \boldsymbol{\lambda}^T \tag{8}$$

In this form, we see three terms: the first shows the drift in the shares' contributions to changes in unit cost due to both price change and technology change; the second term shows the change in unit cost due to change in effective prices modified by the Hessian elasticities *given* prevailing levels of technology; and the third shows the change in unit cost due to technology change ($\lambda_i = \frac{1}{\tau_i}\frac{d\tau_i}{dt}$). More specifically, the second term is (a summation of) $\frac{\partial c}{\partial p_i}\frac{dp_i}{dt}$ and the third is (a summation of) $\frac{\partial c}{\partial \tau_i}\frac{d\tau_i}{dt}$.

Whereas in the Jorgenson et al. approach the number of unknown parameters for each sector is 16 (14 after imposing global concavity constraints), the factor-augmenting approach uses 13 (and 13 after imposing the Ryan & Wales local concavity constraints).

Policy Implications

The most obvious implication of these results, presuming they are deemed credible, is that expectations that energy efficiency gains will provide the global economy a significant and non-welfare-reducing remedy for climate change challenges are likely poorly founded expectations.

[33] Allan et al. (2006, p.17) introduced this concept of "effective" price and demand given technology gain. Here their idea is seen as being perhaps even more broadly useful analytically.

Accordingly, an unfortunate consequence is that climate change remedies must instead rely much more heavily on the development of clean, cheap, abundant energy supplies than has perhaps been appreciated by many energy analysts.

A second, important, implication of the results is that the evident large variation of rebound propensities among sectors means that the application of carbon taxes (or their economic equivalent, auction-based cap-and-trade protocols), to the extent these become necessary, are likely to have far different results in some sectors vs. others. Since lower rebound magnitudes are likely associated with (at least from theoretical considerations) lower substitution elasticities, and since lower substitution elasticities correspond with larger output-reducing effects of a given level of carbon tax, a uniform carbon tax applied across sectors may unfairly penalize some sectors over others.[34] This suggests that any carbon tax policy, to be most effective in delivering energy consumption reductions with least cost to economic welfare, may need to become sector-specific (or even firm-specific). Much work needs to be done to devise carbon taxation policy that is optimally effective and fair.

Finally, policy makers need to become serious about funding research aimed at much more deeply understanding the complex effects of technological efficiency gains on energy consumption. The results presented here merely scratch the surface of a very large boulder representing the real—and critical-to-climate-change-remedies—work that must be done if policy makers are to be adequately informed for sound decision making. But it is arguably a much more substantive "scratch" than is visible in the models relied on by the IPCC.

Conclusion

This analysis appears to present a significant challenge to the methodology used by the IPCC to forecast energy consumption trends, which have a major bearing on setting priorities for climate change policies. But it also lays down some technical challenges to the energy economics community.

Perhaps most urgently, this article can be in part interpreted as an appeal to energy economists to include *measured, flexible* cost/production functions in their analyses of energy consumption trends and rebound magnitudes; and also an appeal to use *measured* technology gains for *all* factors of production.

But the challenge is offered as something larger than this. While the analysis of this article is possibly the most thorough, methodical, systematic, and carefully done analysis of evidence for energy consumption rebound to date, given the cautions and limitations cited above and the enormous stakes involved, it is greatly to be hoped by any serious student of the subject that this analysis is very soon superseded by a much better one.

In the meantime, the toolkit offered here may provide a convenient means for attacking the measurement of rebound in various economies for comparison to the US economy, and, one hopes, a relatively reliable one.

[34] See Saunders (2009).

Note

The toolkit posted online alongside this article is open source and is freely available to any researcher for non-commercial purposes. Toolkit and output files for any sector containing the results reported in this article are available on request from the author. The toolkit includes a detailed User Guide. To accompany the User Guide, media demos of the toolkit are freely available that illustrate its use in a quickly-comprehensible way. These files are too large to post online alongside the article, but can be obtained from the author using file transfer protocol software that is freely available online.

Useful applications of this toolkit would include using it to explore whether a particular economic sector in other countries exhibits different rebound behavior from its counterpart sector in the US economy. A second, important, application would be to test the hypothesis that developing countries exhibit larger rebound magnitudes than industrialized countries. This hypothesis is commonly put forward by energy economists, but thus far is based largely on theoretical considerations and anecdotal evidence. For instance, if the required data exist for India, proving or disproving the hypothesis in this case could have far reaching consequences for climate change policy.

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Appendix A: Methodology Detail

This Appendix contains further detail on the methodology, organized according to the modules of the toolkit provided online. Additional details can be found in the toolkit itself, which is heavily loaded with comments and is accompanied by a User Guide.

Data Conversion Module

For each sector, the Jorgenson data containing the input prices (p_K, p_L, p_E, p_M) and value shares (v_K, v_L, v_E, v_M) are first used to calculate the physical input levels ($X = K, L, E, M$):

$$X = \frac{v_X}{p_X}, \quad X = K, L, E, M \tag{A.9}$$

Actually, these are calculated for each year, so the X of equation (A.9) should be thought of as X^t.

These values of K, L, E, M and Y are used as the *actuals* against which "estimated actuals" (generated by the Rebound Measurement Module) are compared.

Output and factor *capacities* are a different story. Under the philosophy that measured cost/production functions reflect the underlying production possibilities visible to firms contemplating new investment in productive capacity, factor and output capacities must be emulated. This "putty-clay" approach requires a further distinction to be made between the output/factor capacities of the *newest* vintage and those already in place.

For the vintages already in place, it is assumed their production choice from among the possibilities has been fixed by the capital having been put in place when they were new vintages. Thus, factor ratios and factor/output ratios are fixed for the older vintages.[35] The overall magnitude of factor/output capacities is assumed to degrade over time according to a depreciation rate that is input from sectoral data. In this module (and the Rebound Measurement Module), vintage factor/output capacities for each vintage are tracked over time via matrices that account for the measured depreciation.

For each year's new vintage, its factor capacities are determined from Shephard's Lemma (notationally, the distinction between factor/output capacities and factor/output uses is designated by the use of a carat, so that for instance X designates actual factor use and $\hat{X}$ designates factor use capacity). Setting aside capital for the moment,

[35] This is effectively the same as assuming Leontief-type technology for the older vintages in place.

$$\Delta \hat{L}^t = \frac{c^{t-1} s_{\Delta L}^{t-1} \Delta \hat{Y}^t}{p_L^{t-1}}$$
$$\Delta \hat{E}^t = \frac{c^{t-1} s_{\Delta E}^{t-1} \Delta \hat{Y}^t}{p_E^{t-1}} \qquad \text{(A.10)}$$
$$\Delta \hat{M}^t = \frac{c^{t-1} s_{\Delta M}^{t-1} \Delta \hat{Y}^t}{p_M^{t-1}}$$

(This relationship is derived as follows:

Shephard's Lemma with unit cost function has the form for factor X:

$$\frac{\partial c}{\partial p_X} = \frac{X}{Y} \qquad \text{(A.11)}$$

Rearranging:

$$X = \frac{\partial c}{\partial p_X} Y = \frac{c}{p_X} \frac{\partial \ln c}{\partial \ln p_X} Y \qquad \text{(A.12)}$$

But it is also the case that

$$\frac{\partial \ln c}{\partial \ln p_X} = \frac{p_X}{c} \frac{\partial c}{\partial p_X} \qquad \text{(A.13)}$$

Substituting (A.11) into (A.13) gives

$$\frac{\partial \ln c}{\partial \ln p_X} = \frac{p_X}{c} \frac{X}{Y} = s_X \qquad \text{(A.14)}$$

Substituting (A.14) into (A.12) then gives

$$X = \frac{c s_X Y}{p_X} \qquad \text{(A.15)}$$

which is the form appearing in (A.10). This is a general result for any CRS cost/production function.)

Capital is treated differently. Depreciation rates (d) for each sector are obtained, for the US economy, from BEA statistics.[36]

For capital, the module allows four different choices. The idea is that capital formation dynamics will differ in the Rebound Measurement Module as between the 100% rebound case, the zero rebound case and the (estimated) "actual" case. Converting the data for the econometric measurement then must also reflect this flexibility. The four methods are:

Accounting calculation: In this method, $\Delta \hat{K}$ is calculated from the data set as:

$$\Delta \hat{K}^t = K^t - (1-d) K^{t-1} \qquad \text{(A.16)}$$

Revenue reinvestment: In this method, $\Delta \hat{K}$ is calculated as:

[36] These are average depreciation rates over the 1960-2005 time horizon. Source: http://www.bea.gov/national/FA2004/SelectTable.asp.

$$\Delta\hat{K}^t = \left(1-d\right)K^{t-1} + r_{rev}\frac{c^tY^t}{p_K^t} \quad \text{(A.17)}$$

Revenue realized by the sector is the product of output price and output, cY. Revenue is converted to capital quantity terms by dividing this by the price of capital ($\frac{cY}{p_K}$). A revenue reinvestment rate is measured econometrically using equation (A.17) and is multiplied by this quantity ($\Delta K = r_{rev}\frac{cY}{p_K}$). The appeal of this approach is that it mimics the "savings rate" capital formation paradigm of neoclassical growth theory. That is, in a neoclassical growth framework, $\Delta K = sY$. If one were to assume that the ratio $\frac{c}{p_K}$ were fixed (i.e., that capital price stays constant in real terms), the two approaches would be identical.

Cash flow reinvestment: In this method, $\Delta\hat{K}$ is calculated as:

$$\Delta\hat{K}^t = r_{cf}\frac{c^tY^t - \left(p_L^tL^t + p_E^tE^t + p_M^tM\right)}{p_K^t} \quad \text{(A.18)}$$

where r_{cf} is the cash flow reinvestment rate. This equation reflects the sector's revenue less payments to factor inputs for the period (excluding capital payments, since payments to the other factors are obligatory while payments to shareholders are not), converted to capital quantity terms. The parameter r_{cf} is measured econometrically using equation (A.18).

Shephard's Lemma: In this method, $\Delta\hat{K}$ is calculated according to the approach adopted in equation (A.10):

$$\Delta\hat{K}^t = \frac{c^ts_{\Delta K}^t\Delta\hat{Y}^{t-1}}{p_K^t} \quad \text{(A.19)}$$

Then, the new vintage's output capacity is calculated from the accounting equation:

$$\Delta Y^t = \frac{\left(p_K^t\Delta K^t + p_L^t\Delta L^t + p_E^t\Delta E^t + p_M^t\Delta M^t\right)}{c^t} \quad \text{(A.20)}$$

The methods chosen in this article for various sectors are summarized in Appendix C.

These factor/output capacities are added to the factor/output capacities of the existing vintages in each year (corrected for depreciation) to obtain sector-wide factor/output capacities, $\hat{K}^t, \hat{L}^t, \hat{E}^t, \hat{M}^t$. This is done with matrices in the toolkit. Each year's vintages are accounted for by age.

To establish the connection between factor/output capacities and factor/output uses/production, sector-wide utilization rates are calculated for each factor:

$$u_K^t = \frac{K^t}{\hat{K}^t}$$
$$u_L^t = \frac{L^t}{\hat{L}^t}$$
$$u_E^t = \frac{E^t}{\hat{E}^t} \qquad \text{(A.21)}$$
$$u_M^t = \frac{M^t}{\hat{M}^t}$$

These utilizations are transferred to the Rebound Measurement Module.

A subtlety arises for the case of capital utilization. The Jorgenson et al. data set calculates capital quantity and capital price by relying on data reflecting capital in place. But payments to capital should arguably only be considered as going to that capital that is providing capital services—if equipment (and therefore output) is only operating at x% of capacity, the land and structures supporting these capital services being provided should only be considered as supporting them to the extent the equipment is being used. Representing value-producing capital quantity without accounting for this consideration leads to a potential misrepresentation of the derived capital price, which has a significant impact on the econometric measurement of the cost function.

An adjustment to the capital price is therefore called for. This is done as follows:

A simplistic approach would simply adjust the capital price by the same proportion as the capital in place is utilized. However, an additional subtlety arises because of inventories. Inventories provide capital services differently—they are sold at the value of output even when equipment is idle and thus are, in a sense, provided at "full capacity."

Instead, the approach adopted is as follows:

Let $K_{inventory}$ be the "quantity" of capital represented by inventory. Let $K_{other}^{capacity}$ be the quantity of capital in place represented by equipment, structures and land.

Assume that if equipment is utilized at x%, structures and land are also utilized at x%. This utilization rate is reported by the FRB:
http://www.federalreserve.gov/releases/g17/download.htm). Call this rate $u_{equipment}$.

Thus the total capital in place (capital "capacity") is

$$K^{capacity} = K_{other}^{capacity} + K_{inventory} \qquad \text{(A.22)}$$

Total utilized capital is

$$K^{utilized} = u_{equipment} K_{other}^{capacity} + K_{inventory} \qquad \text{(A.23)}$$

From BLS data, US manufacturing-wide, inventories have run on average 12.8% of "produced assets," which are essentially structures and equipment. Call this fraction $f_{inventory}$. Then, from(A.22),

$$K_{inventory} = f_{inventory} K^{capacity} \qquad \text{(A.24)}$$

and

$$K_{other}^{capacity} = \left(1 - f_{inventory}\right) K^{capacity} \tag{A.25}$$

The resulting effective utilization rate for the capital quantities reported by Jorgenson et al is therefore

$$u_{effective} = \frac{K^{utilized}}{K^{capacity}} = \frac{u_{equipment} K_{other}^{capacity} + K_{inventory}}{K_{other}^{capacity} + K_{inventory}} \tag{A.26}$$

Substituting (A.24) and (A.25) into(A.26),

$$u_{effective} = \frac{u_{equipment}\left(1 - f_{inventory}\right) K^{capacity} + f_{inventory} K^{capacity}}{\left(1 - f_{inventory}\right) K^{capacity} + f_{inventory} K^{capacity}} \tag{A.27}$$

or

$$u_{effective} = \frac{u_{equipment}\left(1 - f_{inventory}\right) + f_{inventory}}{\left(1 - f_{inventory}\right) + f_{inventory}} = u_{equipment} + f_{inventory}\left(1 - u_{equipment}\right) \tag{A.28}$$

So one can take the p_K reported by Jorgenson et al., call it $p_K^{reported}$, and adjust it to deliver an adjusted p_K:

$$p_K^{adjusted} = \frac{p_K^{reported}}{u_{effective}} = \frac{p_K^{reported}}{u_{equipment} + f_{inventory}\left(1 - u_{equipment}\right)} \tag{A.29}$$

If, say, the equipment utilization is 84% and we use the BEA average of 12.8% for the inventory fraction, the adjustment becomes

$$p_K^{adjusted} = \frac{p_K^{reported}}{.84 + .128\left(1 - .84\right)} = \frac{p_K^{reported}}{.86} \tag{A.30}$$

This adjustment can be toggled on and off in the Data Conversion Module. However, it is notable that this adjustment almost universally produces better performance of the econometric measurement and lower deviation of projections in the Rebound Measurement Module of factor uses/output compared to actuals.

This solves only part of the utilization problem. Knowing the factor/output utilization rates sector-wide does not reveal the utilization rates across the various vintages, and this will affect the factor uses and output production emulated in the Rebound Measurements Module.

This module allows the user to choose among three different methods:

Uniform utilization: In this method, the assumption is that the sector-wide factor/output utilization rates are applied uniformly across all vintages, meaning each vintage realizes these utilization rates.

Cash flow-determined utilization rates with permanent shutdown: In this method, the assumption is that the sector-wide factor/output utilization rates are reflective of only those vintages experiencing non-negative cash flow in each year. If cash flow is negative for a vintage, it is assumed to be shut down permanently. (Observing the actual behavior of firms, this seems a reasonable method.)

Cash flow-determined utilization rates with temporary shutdown: In this method, like the previous one, the assumption is that the sector-wide factor/output utilization rates are reflective of only those vintages experiencing positive cash flow in each year. But here, if cash flow is negative for a vintage, it is assumed to be sent to standby mode, and is allowed to return to service if factor/output price conditions would result in positive cash flow for that vintage in later years.

As it happens, the *uniform utilization* method almost always delivers the best performance.

In this module, the K, L, E, M and Y values hereby calculated are compared against the actuals. In general, the performance is remarkably good.

With the incremental factor/output capacities so established, these are then converted to the dual world of factor/output prices and factor shares so that the econometric measurement can be done:

$$\begin{aligned} s^t_{\Delta\hat{K}} &= \Delta\hat{K}p_K / \Delta v \\ s^t_{\Delta\hat{L}} &= \Delta\hat{L}p_L / \Delta v \\ s^t_{\Delta\hat{E}} &= \Delta\hat{E}p_E / \Delta v \\ s^t_{\Delta\hat{M}} &= \Delta\hat{M}p_M / \Delta v \end{aligned} \tag{A.31}$$

where $\Delta v = \sum_X \Delta\hat{X}p_X$ is the total of factor value contributions.

The methods chosen in this article for various sectors are summarized in Appendix C.

Econometric Measurement Module

This module takes output from the Data Conversion Module and uses it to econometrically measure the parameters of the cost function.

To generate credible results, analysts usually require that the measured function is concave. It is shown elsewhere (Saunders, 2008) that forcing concavity on a Translog function globally will automatically lead it to predict backfire (rebound greater than 100%). The procedure introduced by Ryan and Wales (2000) overcomes this problem by requiring concavity only locally, although this does not ensure concavity over the period of interest.

Nonetheless, the Ryan and Wales restrictions are the best (and only) option available given use of the Translog. The Ryan and Wales restrictions are typically imposed as follows:

Given a Translog function (in the absence of technology) of the form,

$$\ln c = \ln c_0 + \sum_{i=1}^{N} a_i \ln p_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij} \ln p_i \ln p_j \tag{A.32}$$

Ryan and Wales' procedure calls for replacing the b_{ij} in the estimating equations with the following:

$$b_{ij} = -(DD^T) + a_j\delta_{ij} - a_i a_j \tag{A.33}$$

Where D is triangular, and $\delta_{ij} = 1$ if $i = j$ and $= 0$ otherwise.

If one is working with factors K, L, E, M (where the 4th equation gets dropped to accommodate homogeneity) and the elements of D are d_{ij}, the relevant equations are as follows:

$$b_{kk} = -d_{kk}^2 + a_k - a_k^2$$
$$b_{kl} = -d_{kk}d_{kl} - a_k a_l$$
$$b_{ke} = -d_{kk}d_{ke} - a_k a_e$$
$$b_{ll} = -\left(d_{kl}^2 + d_{ll}^2\right) + a_l - a_l^2$$
$$b_{le} = -\left(d_{kl}d_{ke} + d_{ll}d_{le}\right) - a_l a_e$$
$$b_{ee} = -\left(d_{ke}^2 + d_{le}^2 + d_{ee}^2\right) + a_e - a_e^2$$

In the present case, this procedure must be applied to a Translog that includes factor-augmenting technology gains, where, as described in the Comparison to Jorgenson et al. Methodology section, the technology gains are of the form $\tau_i = e^{\lambda_i t}$, for $i = K, L, E, M$. To assure homogeneity, the four factor share equations are reduced to three via substitution. An equation reflecting changes in the unit cost is added. The resulting four equations are:

$$
\begin{aligned}
s_{\Delta K} &= a_K + b_{KK}\, ln\left(p_K/p_M\right) + b_{KL}\, ln\left(p_L/p_M\right) + b_{KE}\, ln\left(p_E/p_M\right) \\
&\quad -\left[b_{KK}\left(\lambda_K-\lambda_M\right)+b_{KL}\left(\lambda_L-\lambda_M\right)+b_{KE}\left(\lambda_E-\lambda_M\right)\right]t \\
s_{\Delta L} &= a_L + b_{KL}\, ln\left(p_K/p_M\right) + b_{LL}\, ln\left(p_L/p_M\right) + b_{LE}\, ln\left(p_E/p_M\right) \\
&\quad -\left[b_{KL}\left(\lambda_K-\lambda_M\right)+b_{LL}\left(\lambda_L-\lambda_M\right)+b_{LE}\left(\lambda_E-\lambda_M\right)\right]t \\
s_{\Delta E} &= a_E + b_{KE}\, ln\left(p_K/p_M\right) + b_{LE}\, ln\left(p_L/p_M\right) + b_{EE}\, ln\left(p_E/p_M\right) \\
&\quad -\left[b_{KE}\left(\lambda_K-\lambda_M\right)+b_{LE}\left(\lambda_L-\lambda_M\right)+b_{EE}\left(\lambda_E-\lambda_M\right)\right]t \\
d\,ln\,c &= -\left[\lambda_M + a_K\left(\lambda_K-\lambda_M\right)+a_L\left(\lambda_L-\lambda_M\right)+a_E\left(\lambda_E-\lambda_M\right)\right] \\
&\quad +\left[d\,ln\,p_M + a_K\left(d\,ln\,p_K - d\,ln\,p_M\right)+a_L\left(d\,ln\,p_L - d\,ln\,p_M\right)+a_E\left(d\,ln\,p_E - d\,ln\,p_M\right)\right] \\
&\quad +\frac{1}{2}\left\{\begin{array}{l} b_{KK}\left[2\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_K - 2\,ln\,p_K\lambda_K\right] \\ +b_{KL}\left[\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_L+\left(ln\,p_L-\lambda_L t\right)\right]d\,ln\,p_K - ln\,p_K\lambda_L - ln\,p_L\lambda_K \end{array}\right\} \\
&\quad +b_{KE}\left[\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_E+\left(ln\,p_E-\lambda_E t\right)d\,ln\,p_K - ln\,p_K\lambda_E - ln\,p_E\lambda_K\right] \\
&\qquad -\left(b_{KK}+b_{KL}+b_{KE}\right)\left[\begin{array}{l}\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_M+\left(ln\,p_M-\lambda_M t\right)d\,ln\,p_K \\ -ln\,p_K\lambda_M - ln\,p_M\lambda_K\end{array}\right] \\
&\quad +b_{KL}\left[\left(ln\,p_L-\lambda_L t\right)d\,ln\,p_K+\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_L - ln\,p_L\lambda_K - ln\,p_K\lambda_L\right] \\
&\qquad +b_{LL}\left[2\left(ln\,p_L-\lambda_L t\right)d\,ln\,p_L - 2\,ln\,p_L\lambda_L\right] \\
&\quad +b_{LE}\left[\left(ln\,p_L-\lambda_L t\right)d\,ln\,p_E+\left(ln\,p_E-\lambda_E t\right)d\,ln\,p_L - ln\,p_L\lambda_E - ln\,p_E\lambda_L\right] \\
&\qquad -\left(b_{KL}+b_{LL}+b_{LE}\right)\left[\begin{array}{l}\left(ln\,p_L-\lambda_L t\right)d\,ln\,p_M+\left(ln\,p_M-\lambda_M t\right)d\,ln\,p_L \\ -ln\,p_L\lambda_M - ln\,p_M\lambda_L\end{array}\right] \\
&\quad +b_{KE}\left[\left(ln\,p_K-\lambda_E t\right)d\,ln\,p_K+\left(ln\,p_K-\lambda_K t\right)d\,ln\,p_E - ln\,p_K\lambda_E - ln\,p_E\lambda_K\right] \\
&\qquad +b_{LE}\left[\begin{array}{l}\left(ln\,p_E-\lambda_E t\right)d\,ln\,p_L+\left(ln\,p_L-\lambda_L t\right)d\,ln\,p_E \\ -ln\,p_L\lambda_E - ln\,p_E\lambda_L\end{array}\right] \\
&\quad +b_{EE}\left[2\left(ln\,p_E-\lambda_E t\right)d\,ln\,p_E - 2\,ln\,p_E\lambda_E\right] \\
&\qquad -\left(b_{KE}+b_{LE}+b_{EE}\right)\left[\begin{array}{l}\left(ln\,p_E-\lambda_E t\right)d\,ln\,p_M+\left(ln\,p_M-\lambda_M t\right)d\,ln\,p_E \\ -ln\,p_M\lambda_E - ln\,p_E\lambda_M\end{array}\right] \\
&\quad +\frac{1}{2}\left[\begin{array}{l} b_{KK}\left(\lambda_K-\lambda_M\right)+b_{LL}\left(\lambda_L-\lambda_M\right)+b_{EE}\left(\lambda_E-\lambda_M\right)+2b_{KL}\left(\lambda_K-\lambda_M\right)\left(\lambda_L-\lambda_M\right) \\ +2b_{KE}\left(\lambda_K-\lambda_M\right)\left(\lambda_E-\lambda_M\right)+2b_{LE}\left(\lambda_L-\lambda_M\right)\left(\lambda_E-\lambda_M\right)\end{array}\right]t
\end{aligned}
$$

(A34)

The Ryan and Wales constraints are introduced via the following substitutions:

$$
\begin{aligned}
b_{KK} &= d_{KK}^2 + a_K - a_K^2 \\
b_{KL} &= -d_{KK} d_{KL} - a_K a_L \\
b_{KE} &= -d_{KK} d_{KE} - a_K a_E \\
b_{LL} &= -\left(d_{KL}^2 + d_{LL}^2\right) + a_L - a_L^2 \\
b_{LE} &= \left(d_{KL} d_{LE} + d_{LL} d_{LE}\right) - a_L a_E \\
b_{EE} &= -\left(d_{KE}^2 + d_{LE}^2 + d_{EE}^2\right) + a_E - a_E^2
\end{aligned} \qquad \text{(A35)}
$$

The TSP® code implementing these equations in a multi-stage search algorithm is contained in the toolkit provided online alongside this article.

A second component of this module measures, from the same data, parameters relating to capital formation. For determining new capital formation, 4 different methods are available in this module, the *Accounting calculation*, R*evenue investment*, *Cash flow reinvestment*, and *Shephard's Lemma*, described in the Data Conversion Module section. For each sector, the parameters associated with (A.17) and (A.18) are estimated.

Concavity Tests Module

Once the cost function has been econometrically measured, it must be tested year by year to determine if it is concave (or at least quasi-concave) over the domain of interest. It is not sufficient that its core Hessian (the matrix $\mathbf{B}$) be negative semi-definite. Instead, concavity is conveniently tested for using the procedure developed by Diewert and Wales (1987, p.48).

Concavity can be measured by calculating the eigenvalues of a particular matrix that is an adjusted form of the core Hessian. Specifically, Diewert and Wales show that if and only if the following matrix $\mathbf{M}$ is negative (semi-)definite at a particular point, the corresponding cost function will be (quasi-)concave at that point:

$$\mathbf{M} = \mathbf{B} - \hat{\mathbf{s}} + \mathbf{s}\mathbf{s}^T \qquad \text{(A.36)}$$

where $\hat{\mathbf{s}}$ is a matrix with factor (fractional) value shares along the diagonal and zeroes elsewhere, and $\mathbf{s}$ is the vector of (fractional) factor value shares.

The matrix $\mathbf{M}$ will be negative (semi-)definite if and only if all its eigenvalues are (non-positive) negative.

Because this matrix is a function of the factor value shares, the magnitudes of its elements will depend on the prices in any one year. The Concavity Tests Module automates the process of testing for year-by-year concavity with the click of the button. A graphic is delivered showing the fidelity to concavity in each year, and a metric is calculated (the sum of all the largest positive eigenvalues over the domain of interest) that permits a quick assessment of concavity performance. Of course, the module also shows the largest positive eigenvalue in each year if further precision is required.

Rebound Measurement Module

This module is the core of the methodology. Incremental factor capacities for the newest vintage are calculated as in the Data Conversion model [equations (A.10), (A.16), (A.17), (A.18) and (A.19)]. However, while these equations there rely on values for $c^t, s^t_{\Delta X}$ and p^t_X derived from actual data, here they must be calculated for the different rebound scenarios. This is done as follows:

The original data set prices are used as inputs, with the exception of the capital price. For clarity, specify the non-capital prices as

$$^{data}p^t_L,\ ^{data}p^t_E,\ ^{data}p^t_M \quad \text{(A.37)}$$

The capital price is calculated endogenously (and will vary by rebound scenario) from the following equation:

$$p^t_K = \left(\frac{\Delta\hat{Y}^{t-1}}{\Delta\hat{K}^{t-1}}\right) c^{t-1} s^{t-1}_{\Delta K} \quad \text{(A.38)}$$

To calculate the $\Delta\hat{K}^{t-1}$, this module allows the user to choose among all the same investment methods contained in the Data Conversion Module; this determines the incremental capital in any year. Calculation of the $\Delta\hat{Y}^{t-1}$ is explained below, as it is a little more complex. The unit cost and share terms are calculated as follows:

$$c^t = c\left(p^t_K,\ ^{data}p^t_L,\ ^{data}p^t_E,\ ^{data}p^t_M,\ ^{measured}\lambda^t_K,\ ^{measured}\lambda^t_L,\ ^{measured}\lambda^t_E,\ ^{measured}\lambda^t_M\right) \quad \text{(A.39)}$$

and

$$s^t_{\Delta X} = s^t_{\Delta X}\left(p^t_K,\ ^{data}p^t_L,\ ^{data}p^t_E,\ ^{data}p^t_M,\ ^{measured}\lambda^t_K,\ ^{measured}\lambda^t_L,\ ^{measured}\lambda^t_E,\ ^{measured}\lambda^t_M\right) \quad \text{(A.40)}$$

where, for instance, $^{measured}\lambda^t_E$ is the energy technology parameter measured in the econometric estimation.

For the 100% rebound energy-specific different rebound scenario, this procedure is used to create an energy consumption trajectory where $^{measured}\lambda^t_E$ is set to zero, using the measured cost function parameters. The energy consumption trajectories are essentially derived from equations (A.10), (A.17), (A.18) and (A.19) for new vintage factor capacities, along with new vintage output capacities derived as described below, and applying utilization rates applied as *also* described below to develop overall energy demand for all vintages using vintage matrix logic (yet again described below).

For the zero rebound energy-specific different rebound scenario, the only difference from the 100% rebound case is that the new vintage's *energy* capacity is allowed to decline, while keeping other factor and output capacity as in the 100% rebound case, according to a pure "engineering" improvement in energy capacity—energy capacity of the newest vintage is allowed to decline at the pure factor-augmenting rate of $^{measured}\lambda^t_E$. (If this occurs while other factor and output capacity is unchanged, and given that the unit cost is identical to that of the 100% rebound case, this is definitional of Leontief production technology.)

For the "actual" energy trajectory case (which is actually an estimate of the actual), the 100% procedure is followed, but this time with the actual energy technology gain changed from zero to its measured value.

Capital Vintaging Logic

Each vintage's capacity characteristics are added to total capacity in the year it is put in place. The factor/output characteristics are then considered to be established for all time (Leontief Technology assumption). However, over time, the vintage's factor and output capacities are all assumed to depreciate at the rate determined from input data for that sector. This is done with matrices in the toolkit. Each year's vintages are tracked and accounted for by age.

Utilization Logic

To obtain the energy consumption trajectory associated with each rebound case (and also the other factor and output trajectories), utilization rates must be applied to the sector's energy capacity (and must be applied to other factors' capacity and output capacity) to obtain realized consumption of factors and realized production of output. For the estimate of the actual case, historical utilization rates are applied according to the various methods described in the Data Conversion Module section above. For the 100% rebound and zero rebound cases, these utilizations are adjusted according to the methodology described in the following section, ***Application of Cobb-Douglas Consumer Utilities and General Equilibrium.*** The utilization rates change in these other cases because the cost of output changes when changes are made to the technology parameters in equation (A.39), causing consumers to demand more or less output.

Factor Capacities

Factor capacities for the newest vintage are derived in a fashion similar to that used in the Data Conversion Module, with some adjustment to simplify time lags:

$$\begin{aligned}\Delta\hat{L}^t &= \frac{c^t s^t_{\Delta L} \Delta\hat{Y}^{t-1}}{p^t_L} \\ \Delta\hat{E}^t &= \frac{c^t s^t_{\Delta E} \Delta\hat{Y}^{t-1}}{p^t_E} \\ \Delta\hat{M}^t &= \frac{c^t s^t_{\Delta M} \Delta\hat{Y}^{t-1}}{p^t_M}\end{aligned} \tag{A.41}$$

The factor capacity for capital is determined by applying equations identical to equations (A.16) through (A.19).

Output Capacity

While obtaining the output capacity of the newest vintage ($\Delta\hat{Y}^t$) in the Data Conversion Module is a simple case of applying the accounting identity based on calculations of the incremental factor capacities from the data set, (see equation(A.20)), for "backcasting" purposes allowance must be made for the fact that incremental output capacity will in general be different as among the 100% rebound, zero rebound, and actual cases. This module permits application of two basic approaches.

In the first approach, the *Accounting method*, the equivalent of equation (A.20) is used:

$$\Delta\hat{Y}_t = \left(p_K\Delta\hat{K}_t + p_L\Delta\hat{L}_t + p_E\Delta\hat{E}_t + p_M\Delta\hat{M}_t\right)/c_t \tag{A.42}$$

In the second approach, *Ohti's Theorem method*, the procedure is based on an extension of Ohti's Theorem, found in Ohti (1975).

Ohti's Theorem must be extended from its original formulation because it is stated in partial, not full, derivative form:

$$-\frac{\partial\, ln\, c}{c} = \frac{\partial\, ln\, Y}{Y} \tag{A.43}$$

But note that it will ***not***, in general, be true that

$$-\frac{d\, ln\, c}{c} = \frac{d\, ln\, Y}{Y} \tag{A.44}$$

In fact, the Jorgenson data reveal that for most sectors, both Y and c grow together, both with a positive growth rate.

The *Ohti's Theorem method* can be invoked in one of three forms.

Ohti's Theorem Method—Delta X form (prices)

The new vintage's capacity can be represented by a general CRS production function,[37] $\Delta\hat{Y} = \Delta\hat{Y}\left[\Delta\hat{\mathbf{X}}(t), \boldsymbol{\tau}(t)\right]$, where the $\Delta\hat{\mathbf{X}}(t)$ are the factor capacities (quantities) associated with the newest vintage and the $\boldsymbol{\tau}(t)$ are the factor-augmenting technology terms $\tau_K = e^{\lambda_K t}, \tau_L = e^{\lambda_L t}, \tau_E = e^{\lambda_E t}, \tau_K = e^{\lambda_M t}$. (Note that choosing a reference year allows scaling of these terms so that $\tau_X = 1$ at $t = 0$.)

However, since $\boldsymbol{\tau}(t)$ is wholly internal to the function and does not vary independently of t (unlike $\hat{\mathbf{X}}(t)$, which varies independently), the production function can be written equivalently as $\Delta\hat{Y} = \Delta\hat{Y}\left[\Delta\hat{\mathbf{X}}(t), t\right]$.

The derivative of $\Delta\hat{Y}$ with respect to time is then:

[37] In this case, it will be the production function that is dual to the measured Translog cost function (this dual is not a Translog production function—see Saunders, 2008).

$$\frac{d\Delta\hat{Y}}{dt} = \sum_X \frac{\partial\Delta\hat{Y}}{\partial\hat{X}}\frac{d\hat{X}}{dt} + \frac{\partial\Delta\hat{Y}}{\partial t}\frac{dt}{dt} \tag{A.45}$$

[*Note: for notational simplicity, the* t *superscript is dropped in equations* (A.46) *through* (A.76) *for the variables/parameters* c^t, p_X^t, $s_{\Delta X}^t$, $\Delta\hat{Y}^t$, $\Delta\hat{X}^t$, ${}_t\nu_{p_K}^{\Delta\hat{L}}$ *and* σ_{ij}^t *except where explicitly needed*.]

From the first-order condition,

$$\frac{\partial\Delta\hat{Y}}{\partial\Delta\hat{X}} = \frac{p_X}{c} \tag{A.46}$$

Substituting,

$$\frac{1}{\Delta\hat{Y}}\frac{d\Delta\hat{Y}}{dt} = \frac{1}{\Delta\hat{Y}}\sum_X \frac{p_X}{c}\frac{d\Delta\hat{X}}{dt} + \frac{1}{\Delta\hat{Y}}\frac{\partial\Delta\hat{Y}}{\partial t} \tag{A.47}$$

The second term of this expression can be made more tractable. From Ohti's Theorem for the partial,

$$\frac{1}{\Delta\hat{Y}}\frac{\partial\Delta\hat{Y}}{\partial t} = -\frac{1}{c}\frac{\partial c}{\partial t} \tag{A.48}$$

This partial can then be expanded as follows

$$\frac{1}{\Delta\hat{Y}}\frac{\partial\Delta\hat{Y}}{\partial t} = -\frac{1}{c}\frac{\partial c}{\partial t} = -\frac{1}{c}\sum_X \frac{\partial c}{\partial\tau_X}\frac{\partial\tau_X}{\partial t} \tag{A.49}$$

Knowing that

$$\tau_X = e^{\lambda_X t} \tag{A.50}$$

and that[38]

$$\frac{\partial c}{\partial\tau_X} = -\frac{cs_{\Delta X}}{\tau_X} \tag{A.51}$$

this yields

$$\frac{1}{\Delta\hat{Y}}\frac{\partial\Delta\hat{Y}}{\partial t} = -\frac{1}{c}\frac{\partial c}{\partial t} = -\frac{1}{c}\sum_X \frac{\partial c}{\partial\tau_X}\frac{\partial\tau_X}{\partial t} = -\frac{1}{c}\sum_X \left(\frac{-cs_{\Delta X}}{\tau_X}\right)\lambda_X\tau_X = \sum_X s_{\Delta X}\lambda_X \tag{A.52}$$

Thus, from (A.47)

$$\eta_t^{\Delta\hat{Y}} = \frac{1}{\Delta\hat{Y}}\frac{\partial\Delta\hat{Y}}{\partial t} = \frac{1}{\Delta\hat{Y}}\sum_X \frac{p_X}{c}\frac{d\Delta\hat{X}}{dt} + \sum_X s_{\Delta X}\lambda_X \tag{A.53}$$

This gives the first method for calculating the output capacity. Note that in each period the $\Delta\hat{X}_t$ are calculated from Shephard's Lemma. From the econometric measurement, unit cost c, and λ_X are given for each time period. Further, the p_X are available from the data. This gives everything needed to calculate the output capacity of the newest vintage for this form of the "Ohti's Theorem Method":

---

[38] Derived in Saunders (2008), Proofs Appendix F (Lemma 1).

$$\Delta\hat{Y}^{t} = \Delta\hat{Y}^{t-1}e^{\eta_t^Y} = \Delta\hat{Y}^{t-1}e^{\frac{1}{\Delta\hat{Y}^{t-1}}\sum_X \frac{p_X}{c}\frac{d\Delta\hat{X}}{dt} + \sum_X s_{\Delta X}\lambda_X} \quad \text{(A.54)}$$

This form allows the expression of the time elasticity of $\Delta\hat{Y}$ in terms of other time elasticities. Letting

$$\eta_t^{\Delta\hat{Y}} = \frac{1}{\Delta\hat{Y}}\frac{d\Delta\hat{Y}}{dt}; \quad \eta_t^{\Delta\hat{X}} = \frac{p_X}{c}\frac{1}{\Delta\hat{Y}}\frac{d\Delta\hat{X}}{dt}; \quad \eta_t^{\tau} = \sum_X s_{\Delta X}\lambda_X \quad \text{(A.55)}$$

yields the relationship

$$\eta_t^{\Delta Y} = \eta_t^{\Delta K} + \eta_t^{\Delta L} + \eta_t^{\Delta E} + \eta_t^{\Delta M} + \eta_t^{\tau} \quad \text{(A.56)}$$

This gives a compact way to look at the contributions to output growth from each factor and from technology gains. η_t^{τ} can be further decomposed into its individual factor elasticities:

$$\eta_t^{\tau} = \sum_X \eta_t^{\tau_X}, \quad \eta_t^{\tau_X} = s_{\Delta X}\lambda_X \quad \text{(A.57)}$$

Ohti's Theorem Method—Delta X form (shares)

In a second method, a further substitution can be made by observing that

$$\frac{\partial \ln c}{\partial \ln p_X} = \frac{p_X}{c}\frac{\partial c}{\partial p_X} = \frac{p_X}{c}\frac{\Delta\hat{X}}{\Delta\hat{Y}} = s_{\Delta X} \quad \text{(A.58)}$$

yielding

$$\frac{p_X}{c} = \frac{\Delta\hat{Y}}{\Delta\hat{X}}s_{\Delta X} \quad \text{(A.59)}$$

So making these substitutions and dividing through by $\Delta\hat{Y}$, (A.47) becomes

$$\eta_t^{\Delta\hat{Y}} = \frac{1}{\Delta\hat{Y}}\frac{d\Delta\hat{Y}}{dt} = \sum_X s_{\Delta X}\frac{1}{\Delta\hat{X}}\frac{d\Delta\hat{X}}{dt} + \sum_X s_{\Delta X}\lambda_X \quad \text{(A.60)}$$

This form allows the time elasticity of $\Delta\hat{Y}$ to be expressed in terms of other time elasticities. Letting

$$\eta_t^{\Delta\hat{Y}} = \frac{1}{\Delta\hat{Y}}\frac{d\Delta\hat{Y}}{dt}; \quad \eta_t^{\Delta\hat{X}} = s_{\Delta X}\frac{1}{\Delta\hat{X}}\frac{d\Delta\hat{X}}{dt}; \quad \eta_t^{\tau} = \sum_X s_{\Delta X}\lambda_X \quad \text{(A.61)}$$

yields the relationship

$$\eta_t^{\Delta\hat{Y}} = \eta_t^{\Delta\hat{K}} + \eta_t^{\Delta\hat{L}} + \eta_t^{\Delta\hat{E}} + \eta_t^{\Delta\hat{M}} + \eta_t^{\tau} \quad \text{(A.62)}$$

This again gives a compact way to look at the contributions to output growth from each factor and from technology gains. Again, η_t^{τ} can be further decomposed into its individual factor elasticities:

$$\eta_t^{\tau} = \sum_X \eta_t^{\tau_X}, \quad \eta_t^{\tau_X} = s_{\Delta X}\lambda_X \quad \text{(A.63)}$$

Further, it is straightforward to develop "technology-enhanced" elasticity measures for each measure:

$$\eta_t^{\Delta \hat{X}_\tau} = s_{\Delta X}\left(\frac{1}{\Delta \hat{X}}\frac{d\Delta \hat{X}}{dt} + \lambda_X\right) \tag{A.64}$$

where the τ subscripts distinguish the $\Delta \hat{X}$ superscripts from the $\Delta \hat{X}_\tau$ superscripts.

This yields

$$\eta_t^{\Delta \hat{Y}} = \eta_t^{\Delta \hat{K}_\tau} + \eta_t^{\Delta \hat{L}_\tau} + \eta_t^{\Delta \hat{E}_\tau} + \eta_t^{\Delta \hat{M}_\tau} \tag{A.65}$$

Equation (A.60) can therefore also be written

$$\eta_t^{\Delta \hat{Y}} = \frac{1}{\Delta \hat{Y}}\frac{d\Delta \hat{Y}}{dt} = \sum_X s_{\Delta X}\left(\frac{1}{\Delta \hat{X}}\frac{d\Delta \hat{X}}{dt} + \lambda_X\right) \tag{A.66}$$

As with the first method, note that in each period the $\Delta \hat{X}_t$ are calculated from Shephard's Lemma. The econometric measurement delivers the $s_{\Delta X}$ and the λ_X in each time period. This provides everything needed to calculate the output capacity of the newest vintage, from (A.66) for this form of "Ohti's Theorem Method"):

$$\Delta \hat{Y}^t = \Delta \hat{Y}^{t-1} e^{\eta_t^{\Delta Y}} = \Delta \hat{Y}^{t-1} e^{\sum_{\Delta X} s_{\Delta X}\frac{1}{\hat{X}^{t-1}}\frac{d\Delta \hat{X}}{dt} + \sum_X s_{\Delta X}\lambda_X} \tag{A.67}$$

The difference between these first two methods has to do with the variable used from the previous period. In the first method, the calculated elasticity relies on the previous period's new vintage output capacity; in the second method, it relies on previous period's new vintage factor capacities.

Ohti's Theorem Method—Price elasticity form

The third method uses factor price elasticities derived from factor substitution elasticities that are automatically calculated year-by-year in the module.

The approach is to calculate the $\Delta \hat{X}$ terms of equation (A.53) differently. To illustrate by using $\Delta \hat{L}$, its expansion yields

$$d\Delta \hat{L} = \frac{\partial \Delta \hat{L}}{\partial p_K}dp_K + \frac{\partial \Delta \hat{L}}{\partial p_L}dp_L + \frac{\partial \Delta \hat{L}}{\partial p_E}dp_E + \frac{\partial \Delta \hat{L}}{\partial p_M}dp_M \tag{A.68}$$

Define the following price elasticity parameters:

$$\nu_{p_K}^{\Delta\hat{L}} = \frac{\partial\Delta\hat{L}}{\partial p_K}\frac{p_K}{d\Delta\hat{L}}$$
$$\nu_{p_L}^{\Delta\hat{L}} = \frac{\partial\Delta\hat{L}}{\partial p_L}\frac{p_L}{d\Delta\hat{L}}$$
$$\nu_{p_E}^{\Delta\hat{L}} = \frac{\partial\Delta\hat{L}}{\partial p_E}\frac{p_E}{d\Delta\hat{L}} \qquad \text{(A.69)}$$
$$\nu_{p_M}^{\Delta\hat{L}} = \frac{\partial\Delta\hat{L}}{\partial p_M}\frac{p_M}{d\Delta\hat{L}}$$

Substituting (A.69) in (A.68) yields:

$$d\Delta\hat{L} = \frac{d\Delta\hat{L}}{p_K}\nu_{p_K}^{\Delta\hat{L}}dp_K + \frac{d\Delta\hat{L}}{p_L}\nu_{p_L}^{\Delta\hat{L}}dp_L + \frac{d\Delta\hat{L}}{p_E}\nu_{p_E}^{\Delta\hat{L}}dp_E + \frac{d\Delta\hat{L}}{p_M}\nu_{p_M}^{\Delta\hat{L}}dp_M \qquad \text{(A.70)}$$

To avoid circular references, this equation can be made time dependent by introducing a lag:

$$d\Delta\hat{L}^t = d\Delta\hat{L}^{t-1}\mathbf{v}_{\mathbf{t}}^{\mathbf{\Delta\hat{L}}}\mathbf{dlnp_K}^T \qquad \text{(A.71)}$$

where $\mathbf{v}_t^{\mathbf{\Delta\hat{L}}} = \left({}_t\nu_{p_K}^{\Delta\hat{L}}, {}_t\nu_{p_L}^{\Delta\hat{L}}, {}_t\nu_{p_E}^{\Delta\hat{L}}, {}_t\nu_{p_M}^{\Delta\hat{L}}\right)_t$.

Putting this result in elasticity form and extending to include factors E and M:

$$\eta_t^{\Delta\hat{L}} = \frac{p_L}{c}\frac{1}{\Delta\hat{Y}^{t-1}}d\Delta\hat{L}^{t-1}\mathbf{v}_{\mathbf{t}}^{\mathbf{\Delta\hat{L}}}\mathbf{dlnp}^T + s_{\Delta\hat{L}}\lambda_L$$
$$\eta_t^{\Delta\hat{E}} = \frac{p_E}{c}\frac{1}{\Delta\hat{Y}^{t-1}}d\Delta\hat{E}^{t-1}\mathbf{v}_{\mathbf{t}}^{\mathbf{\Delta\hat{E}}}\mathbf{dlnp}^T + s_{\Delta\hat{E}}\lambda_E \qquad \text{(A.72)}$$
$$\eta_t^{\Delta\hat{M}} = \frac{p_M}{c}\frac{1}{\Delta\hat{Y}^{t-1}}d\Delta\hat{M}^{t-1}\mathbf{v}_{\mathbf{t}}^{\mathbf{\Delta\hat{M}}}\mathbf{dlnp}^T + s_{\Delta\hat{M}}\lambda_M$$

where $dt = 1$, since the elasticity is measured over one time period.

Capital is treated slightly differently. Because capital formation is combined with an endogenously-determined price of capital, there is no price elasticity effect that influences it (beyond the requirement of conformance to Shephard's Lemma in the method where incremental capital is so determined). Accordingly, the equivalent equation for capital's elasticity contribution to output elasticity is

$$\eta_t^{\Delta\hat{K}} = \frac{p_K}{c}\frac{1}{\Delta\hat{Y}^{t-1}}d\Delta\hat{K}^{t-1} + s_{\Delta\hat{K}}\lambda_K \qquad \text{(A.73)}$$

So the equivalent of (A.65) becomes

$$\eta_t^{\Delta\hat{Y}} = \eta_t^{\Delta\hat{K}_\tau} + \eta_t^{\Delta\hat{L}_\tau} + \eta_t^{\Delta\hat{E}_\tau} + \eta_t^{\Delta\hat{M}_\tau} \qquad \text{(A.74)}$$

where the component elasticities are those given in (A.72) and (A.73).

The price elasticity parameters of (A.72) can be derived from factor shares and the calculated factor substitution elasticities:[39]

$$\nu_{p_K}^{\Delta\hat{L}} = s_{\Delta j}\sigma_{ij} \qquad \text{(A.75)}$$

[39] Frondel (2010).

For a Translog function, the factor substitution elasticities are given as[40]

$$\begin{aligned} \sigma_{ij} &= \frac{b_{ij}}{s_i s_j} + 1 \quad i = j \\ \sigma_{ii} &= \frac{b_{ii}}{s_i^2} - \frac{1}{s_i} + 1 \quad i \neq j \end{aligned} \tag{A.76}$$

The b_{ij} are known from the econometric measurement of the cost function; the s_i are tracked year-by year.

For all three methods, $\Delta\hat{Y}$ is determined as

$$\Delta\hat{Y}^t = \left(\Delta\hat{Y}^{t-1}\right)^{\eta_t^{\Delta\hat{Y}}} \tag{A.77}$$

All these methods generate projections of changing overall output capacity and overall factor capacities that are in tight conformance with economic theory. They are not arbitrarily structured.

Further, these methods, particularly the Price elasticity form of the Ohti's Theorem Method, generate forecasts that closely match actuals. This can be taken as a strong statement of the power of using theoretically sound methods and also seems to provide surprising evidence for the validity of the underlying assumption of perfectly competitive profit-maximizing producers.

The methods chosen in this article for various sectors are summarized in Appendix C.

Application of Cobb-Douglas Consumer Utilities and General Equilibrium

In the 100% rebound case, technology parameters are set to zero (λ_E is set to zero in the "energy-specific rebound" case; all λs are set to zero in the "all factors rebound" case) and this obviously affects the unit cost, which is calculated endogenously. A different price of output will mean that consumer demand for the output from that sector will be changed.

The approach of this article, which facilitates easy aggregation across sectors, is to assume that consumer utilities across sector outputs honor a Cobb-Douglas formulation. In such a case, resulting consumer demand for these outputs can be treated independently.

A Cobb-Douglas utility formulation across n products (sectors) is specified according to the following function:

$$u(\mathbf{x}) = \prod_{i=1}^{n} x_i^{\alpha_i} \tag{A.78}$$

If $\alpha = \sum_{i=1}^{n} \alpha_i$, the demand function for product (sector output) i is[41]

[40] Kuroda et al. (2000), in Jorgenson (2000), pp. 392-393.
[41] Luenberger (1995), p. 132.

$$x_i(\mathbf{p}, r) = \frac{\alpha_i r}{\alpha p_i} \tag{A.79}$$

where $\mathbf{p}$ is the vector of all product (sector output) prices, and r is the consumer budget constraint. The budget constraint is assumed fixed across rebound scenarios.[42]

From this it can be seen that consumer demand for the output of a particular sector is dependent only on the price of that sector's output. This means the elasticity of demand for the sector's output with respect to that sector's output price is

$$\nu_{p_i}^{Y_i} = \frac{p_i}{Y_i}\frac{\partial Y_i}{\partial p_i} = -\frac{p_i}{\alpha}\frac{\alpha}{p_i} = -1 \tag{A.80}$$

If c_A^t is the unit cost at time t associated with the estimated actual case for a particular sector and $c_{100\%}^t$ is the unit cost at time t associated with the 100% rebound case for a particular sector, then demand for output from that sector Y^t can be modified according to the following factor:

$$f^t = -\frac{\left(c_{100\%}^t - c_A^t\right)}{c_A^{t-1}} \tag{A.81}$$

This is actually done indirectly. The factor f^t is applied to the sector-wide utilization rates for each factor (irrespective of the method used to spread utilization across vintages), thus modifying actual factor uses. This has the effect of reducing actual output (as distinct from output capacity) by this same factor since actual output is a linear combination of actual factor uses.

Aggregation across sectors is facilitated by this approach. Different rebound cases have different trajectories of output cost since different technology gains apply to them, thus changing consumer demand for that output. But from (A.79) the outputs in each different rebound case can be simply added together and maintain consistency with general equilibrium.

Likewise, energy use can also be aggregated across sectors for the different rebound cases since energy is assumed to be supplied perfectly elastically. So even though energy quantities differ among sectors as between the rebound cases, the energy market will always clear at the specified energy price. The same holds true for labor and materials markets, since these are also assumed to be supplied elastically.

The capital market is a little different, since the price of capital is determined endogenously given the capital formation mechanism and will differ as between rebound cases. This means the price of capital will vary among sectors. But this is what is actually observed in the Jorgenson et al. data—capital price is highly variable across sectors (unlike other factor prices) and is quite volatile.

So the assumption of Cobb-Douglas consumer utility and the assumption of perfect elasticity of supply for labor, energy and materials provides what can be considered a "poor man's" general equilibrium model where all factor and output markets clear at the specified (or calculated) prices.

[42] See footnote 27.

Appendix B: Analysis of Rebound Time Dynamics

In response to informal peer review suggestions, this Appendix describes in further detail the time dynamics of the rebound effect evident in Figures 1 and 2 of the main text. Rebound time dynamics depend on complex interactions among several determining elements. However, three elements are key: one, the inherent rebound characteristics of the newest production vintage; two, actual technology gains for the newest vintage; and three, the consumer response to changing output price.

Each of these elements is examined in turn. Examining the first requires a partial equilibrium analysis of the factor use capacities of the newest vintage, and specifically an analysis of what determines their dynamic evolution.

Partial Equilibrium Analysis of New Vintage Capacity Rebound

Extending the basic methodology outlined in Saunders (2005, 2008) for a three-factor cost function, the partial equilibrium rebound equations can be derived for a the four-factor CRS cost function used here. This analysis considers only the energy-specific factor technology gain; the corresponding derivation for all factor technology gains is extremely complex and, frankly, has not been attempted.

The analysis begins with a general statement of Shephard's Lemma that honors the factor assumptions of the current article.[43] Specifically, assuming fixed nominal prices for labor, energy, and materials, and that capital quantity $K = K_0$ is specified by one of the four methods described in equations (A.16) through (A.19), Shephard's Lemma becomes:

$$X(\tau) = Y\left[K_0, L(\tau), E(\tau), M(\tau), \tau\right] \frac{\partial c\left[p_K(\tau), p_L^0, p_E^0, p_M^0, \tau\right]}{\partial p_X} \tag{B.1}$$

where $X = L, E, M$ are the factor capacities of the newest vintage and Y is its output capacity.

Differentiating with respect to the energy technology gain τ delivers three equations, one for each of the variable factors:

$$\frac{\partial E(\tau)}{\partial \tau} = \left(\frac{\partial Y}{\partial E}\frac{\partial E}{\partial \tau} + \frac{\partial Y}{\partial L}\frac{\partial L}{\partial \tau} + \frac{\partial Y}{\partial M}\frac{\partial M}{\partial \tau} + \frac{\partial Y}{\partial \tau}\right)\frac{\partial c}{\partial p_E} + Y\left[\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_E}\right)\frac{\partial p_K}{\partial \tau} + \frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_E}\right)\right] \tag{B.2}$$

$$\frac{\partial L(\tau)}{\partial \tau} = \left(\frac{\partial Y}{\partial E}\frac{\partial E}{\partial \tau} + \frac{\partial Y}{\partial L}\frac{\partial L}{\partial \tau} + \frac{\partial Y}{\partial M}\frac{\partial M}{\partial \tau} + \frac{\partial Y}{\partial \tau}\right)\frac{\partial c}{\partial p_L} + Y\left[\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_L}\right)\frac{\partial p_K}{\partial \tau} + \frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_L}\right)\right] \tag{B.3}$$

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[43] For notational tractability in this appendix, the incremental notation designating new vintage characteristics is dropped. Specifically, here we use K, L, M and Y in place of $\Delta\hat{K}$, $\Delta\hat{L}$, $\Delta\hat{E}$ and $\Delta\hat{Y}$, and s_X in place of $s_{\Delta X}$ to indicate incremental shares, but it should be borne in mind that the analysis is applied to the factor and output capacity characteristics of the newest vintage. Further, τ is used in place of τ_E.

$$\frac{\partial M(\tau)}{\partial \tau}=\left(\frac{\partial Y}{\partial E}\frac{\partial E}{\partial \tau}+\frac{\partial Y}{\partial L}\frac{\partial L}{\partial \tau}+\frac{\partial Y}{\partial M}\frac{\partial M}{\partial \tau}+\frac{\partial Y}{\partial \tau}\right)\frac{\partial c}{\partial p_M}+Y\left[\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_M}\right)\frac{\partial p_K}{\partial \tau}+\frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_M}\right)\right] \tag{B.4}$$

Multiplying through each equation by $\frac{\tau}{X}$ and noting that $\frac{\partial Y}{\partial X}=\frac{p_x}{c}$, and $\frac{\partial c}{\partial p_X}=\frac{X}{Y}$, and designating $\eta_\tau^X=\frac{\tau}{X}\frac{\partial X}{\partial \tau}$, $\phi_X=\frac{\tau}{\partial c/\partial p_X}\left[\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_X}\right)\frac{\partial p_K}{\partial \tau}+\frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_X}\right)\right]$, and s_X as the value share of factor X these equations become:

$$(1-s_E)\eta_\tau^E=s_L\eta_\tau^L+s_M\eta_\tau^M+s_E+\phi_E \tag{B.5}$$

$$(1-s_L)\eta_\tau^L=s_E\eta_\tau^E+s_M\eta_\tau^M+s_E+\phi_L \tag{B.6}$$

$$(1-s_M)\eta_\tau^M=s_E\eta_\tau^E+s_L\eta_\tau^L+s_E+\phi_M \tag{B.7}$$

where η_τ^E, η_τ^L, and η_τ^M are the rebound elasticities of factor uses with respect to τ. As defined in Saunders (2005, 2008), energy rebound is then $R=1+\eta_\tau^E$. (Note that these equations illustrate that labor and materials are also subject to their own rebound due to the energy-specific efficiency gain τ.) This system of equations can be solved for η_τ^E by applying Cramer's Rule:

$$\eta_\tau^E=\frac{\begin{vmatrix} s_E+\phi_E & -s_L & -s_M \\ s_E+\phi_L & 1-s_L & -s_M \\ s_E+\phi_M & -s_L & 1-s_M \end{vmatrix}}{\begin{vmatrix} 1-s_E & -s_L & -s_M \\ -s_E & 1-s_L & -s_M \\ -s_E & -s_L & 1-s_M \end{vmatrix}} \tag{B.8}$$

yielding

$$\eta_\tau^E=\frac{s_E+(s_K+s_E)\phi_E+s_L\phi_L+s_M\phi_M}{s_K} \tag{B.9}$$

Temporarily adopting the notation that $\frac{\partial c}{\partial i}=c_i$ and $\frac{\partial}{\partial i}\left(\frac{\partial c}{\partial j}\right)=c_{ij}$, the ϕ_E, ϕ_L and ϕ_M can be simplified by noting that $\frac{\partial c}{\partial p_X}=\frac{cs_X}{p_X}$, $\frac{\partial c}{\partial \tau}=-\frac{cs_E}{\tau}$, and that pair-wise substitution elasticities are $\sigma_{ij}=\frac{cc_{ij}}{c_ic_j}$.[44]

Starting with ϕ_E, we have

[44] See Saunders (2008), Appendix D, pp. 2216-17.

$$\phi_E = \frac{\tau}{\partial c / \partial p_E}\left[\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_E}\right)\frac{\partial p_K}{\partial \tau} + \frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_E}\right)\right] \tag{B.10}$$

Then, beginning with the first term of this expression and making the appropriate substitutions

$$\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_E}\right) = c_{KE} = \frac{c_K c_E \sigma_{KE}}{c} = \frac{c s_K s_E \sigma_{KE}}{p_K p_E} \tag{B.11}$$

so that

$$\frac{\tau}{\partial c / \partial p_E}\frac{\partial}{\partial p_K}\left(\frac{\partial c}{\partial p_E}\right)\frac{\partial p_K}{\partial \tau} = \frac{\tau p_E}{c s_E}\frac{c s_K s_E \sigma_{KE}}{p_K p_E}\frac{\partial p_K}{\partial \tau} = s_K \sigma_{KE}\frac{\tau}{p_K}\frac{\partial p_K}{\partial \tau} = s_K \sigma_{KE}\eta_\tau^{p_K} \tag{B.12}$$

The second term of (B.10) is

$$\frac{\tau}{\partial c / \partial p_E}\frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_E}\right) = \frac{\tau}{c_E}c_{\tau E} \tag{B.13}$$

yielding

$$\phi_E = s_K \sigma_{KE}\eta_\tau^{p_K} + \frac{\tau}{c_E}c_{\tau E} \tag{B.14}$$

Parallel derivations give

$$\phi_L = s_K \sigma_{KL}\eta_\tau^{p_K} + \frac{\tau}{c_L}c_{\tau L} \tag{B.15}$$

$$\phi_M = s_K \sigma_{KM}\eta_\tau^{p_K} + \frac{\tau}{c_M}c_{\tau M} \tag{B.16}$$

Solving for $\eta_\tau^{p_K}$ requires appealing to the cost-constrained profit maximizing formulation described in Saunders (2005, 2008). Specifically, it can be shown that the producer profit maximizing problem can be put in the form:

$$\begin{aligned}&\max_{L,E,M,p_K} \pi = c_0 Y\left(K_0, L, E, M, \tau\right) - p_K K_0 - p_L^0 L - p_E^0 E - p_M^0 M \\ &s.t. \quad c\left(p_K, p_L^0, p_E^0, p_M^0, \tau\right) = c_0\end{aligned} \tag{B.17}$$

Differentiating the Lagrangian of this problem with respect to p_K and the Lagrange multiplier μ yields two equations:[45]

$$\begin{aligned}&-K_0 + \mu\frac{\partial c}{\partial p_K} = 0 \\ &c - c_0 = 0\end{aligned} \tag{B.18}$$

Designating the first of these equations ψ_1 and the second ψ_2, application of the Implicit Function Theorem yields

[45] The remaining equations of this system fall out and are not needed.

$$\begin{bmatrix} \dfrac{\partial p_K}{\partial \tau} \\ \dfrac{\partial \mu}{\partial \tau} \end{bmatrix} = -J^{-1} \begin{bmatrix} \dfrac{\partial \psi_1}{\partial \tau} \\ \dfrac{\partial \psi_2}{\partial \tau} \end{bmatrix} \tag{B.19}$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \dfrac{\partial \psi_1}{\partial p_K} & \dfrac{\partial \psi_1}{\partial \mu} \\ \dfrac{\partial \psi_2}{\partial p_K} & \dfrac{\partial \psi_2}{\partial \mu} \end{bmatrix} = \begin{bmatrix} \mu c_{KK} & c_K \\ c_K & 0 \end{bmatrix} \tag{B.20}$$

Rewriting (B.19) gives

$$\begin{bmatrix} \dfrac{\partial p_K}{\partial \tau} \\ \dfrac{\partial \mu}{\partial \tau} \end{bmatrix} = -J^{-1} \begin{bmatrix} \dfrac{\partial \psi_1}{\partial \tau} \\ \dfrac{\partial \psi_2}{\partial \tau} \end{bmatrix} = \frac{1}{c_{KK}} \begin{bmatrix} 0 & -c_K \\ -c_K & \mu c_{KK} \end{bmatrix} \begin{bmatrix} \mu c_{\tau K} \\ c\tau \end{bmatrix} \tag{B.21}$$

yielding

$$\frac{\partial p_K}{\partial \tau} = -\frac{c_\tau}{c_K} \tag{B.22}$$

Noting that $c_\tau = -\dfrac{cs_E}{\tau}$ and $c_K = \dfrac{cs_K}{p_K}$ and multiplying (B.22) by $\dfrac{\tau}{p_K}$ yields the required result:

$$\eta_\tau^{p_K} = \frac{s_E}{s_K} \tag{B.23}$$

All that remains is to make explicit the partials in equations (B.14) through (B.16). We begin with $c_{\tau E}$:

$$\begin{aligned} c_{\tau E} &= \frac{\partial}{\partial \tau}\left(\frac{\partial c}{\partial p_E}\right) = \frac{\partial}{\partial \tau}\left(\frac{cs_E}{p_E}\right) = \frac{1}{p_E}\frac{\partial}{\partial \tau}\left(cs_E\right) = \frac{1}{p_E}\left(s_E \frac{\partial c}{\partial \tau} + c\frac{\partial s_E}{\partial \tau}\right) \\ &= \frac{1}{p_E}\left(-s_E \frac{cs_E}{\tau} + c\frac{\partial s_E}{\partial \tau}\right) \end{aligned} \tag{B.24}$$

To this point, the derivation is completely general for any CRS cost function. However, the second term of (B.24) requires a specific cost function, in this case the Translog function described in the main text, equations (3) and (4). The energy share is derived by differentiating the Translog function with respect to energy price:

$$s_E = \frac{\partial\, ln\, c}{\partial\, ln\, p_E} = a_E + b_{KE}\, ln\, p_K + b_{LE}\, ln\, p_L + b_{EE}\, ln\left(\frac{p_E}{\tau}\right) + b_{EM}\, ln\, p_M \tag{B.25}$$

so that

$$\frac{\partial s_E}{\partial \tau} = -\frac{b_{EE}}{\tau} \tag{B.26}$$

Recalling that $\sigma_{EE} = \frac{b_{EE}}{s_E^2} - \frac{1}{s_E} + 1$, substituting (B.26) into (B.24) delivers

$$c_{\tau E} = -\frac{cs_E}{p_E \tau}\left(s_E \sigma_{EE} + 1\right) \tag{B.27}$$

Comparable derivations yield:

$$\begin{aligned} c_{\tau L} &= -\frac{cs_L s_E \sigma_{LE}}{p_L \tau} \\ c_{\tau M} &= -\frac{cs_M s_E \sigma_{ME}}{p_M \tau} \end{aligned} \tag{B.28}$$

Substituting (B.27) and (B.28) into (B.14) through (B.16):

$$\begin{aligned} \phi_E &= s_E\left(\sigma_{KE} - \sigma_{EE}\right) - 1 \\ \phi_L &= s_E\left(\sigma_{KL} - \sigma_{LE}\right) \\ \phi_M &= s_E\left(\sigma_{KM} - \sigma_{KE}\right) \end{aligned} \tag{B.29}$$

Finally, substituting (B.29) into (B.9):

$$\eta_\tau^E = \frac{s_E}{s_K}\left\{1 + \left(s_K + s_E\right)\left[\left(\sigma_{KE} - \sigma_{EE}\right) - 1\right] + s_L\left(\sigma_{KL} - \sigma_{LE}\right) + s_M\left(\sigma_{KM} - \sigma_{ME}\right)\right\} \tag{B.30}$$

Note that rebound elasticity depends on all factor shares and pair-wise elasticities of substitution, both of which change dynamically over time. In fact, since the substitution elasticities are determined solely by shares and the fixed, measured parameters of the cost function for any sector, the dynamic evolution of the rebound elasticity depends solely on evolution of factor shares.

Recall that this analysis applies only to the factor and output capacity characteristics of the newest vintage, so the shares in equation (B.30) need to be thought of as the incremental sector shares associated with the newest vintage. Actual rebounds in any year will be determined by this along with other elements to be described shortly. This element, the rebound elasticity of equation (B.30), can be thought of as the "inherent" rebound propensity of the newest vintage of productive capacity for each sector.

The toolkit provided alongside this article automatically calculates this annual inherent rebound propensity for any sector. The figure below illustrates this for a few selected sectors that are representative of the surprising range of rebound dynamics (recalling that instantaneous rebound $R = 1 + \eta_\tau^E$):

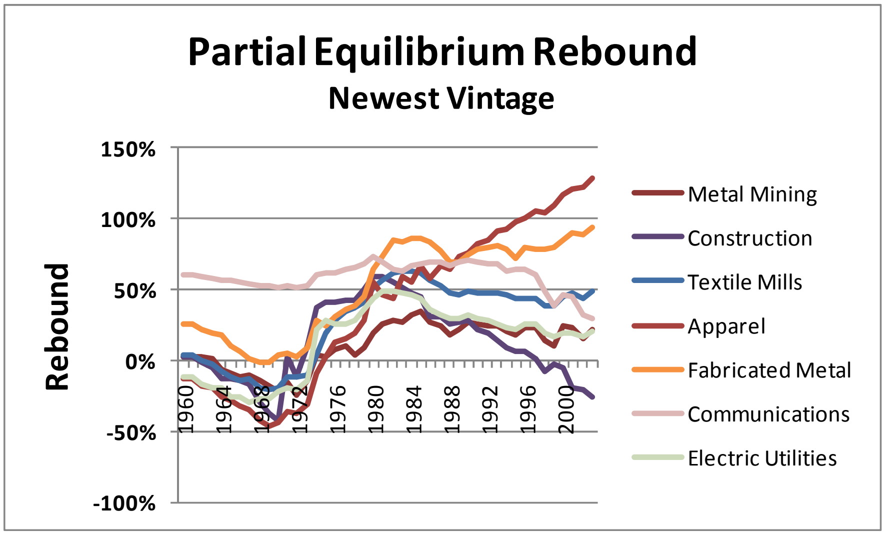

Figure B-1. Inherent Rebound Propensity for Selected Sectors (Energy technology gain only)

Note that there appears to be a certain periodicity for some sectors,[46] and that the dynamic evolution can vary dramatically as between sectors. Note also that "super-conservation"—the possibility of greater than one-for-one reductions in energy consumption with energy efficiency gain—is not disallowed, at least for a time.[47] However, in the periods showing such negative rebound, concavity performance is generally relatively poor.

Actual Rebound Characteristics of New Vintage Capacity

The preceding analysis describes the rebound propensity of the newest vintage for a unit change in energy efficiency. The actual rebound characteristics of the new vintage will depend on the magnitude of the energy efficiency gain, measured econometrically for each sector. A larger efficiency gain will, *ceteris paribus*, result in a greater fraction or multiple of the inherent rebound magnitude being realized.

Further, even if the inherent rebound propensity is negative, a larger efficiency gain will result in a greater output component of rebound being realized, and the output component cumulates over time. The output effect is compounded by an increase in the capital quantity available for the new vintage. That is, whereas the preceding analysis considered capital quantity fixed in any one year, determined by the capital formation method chosen from among the four methods described in equations (A.16) through (A.19), in actuality the capital quantity will grow over time given an energy efficiency gain (relative to an absence of this gain). The figure below illustrates the difference.

[46] This dynamic is not due to model-based lags. All inputs to equation (B.30) each year are from the same year and rely on the same data year.
[47] See Saunders (2005) for more discussion of this rebound condition as allowed by different production/cost function specifications.

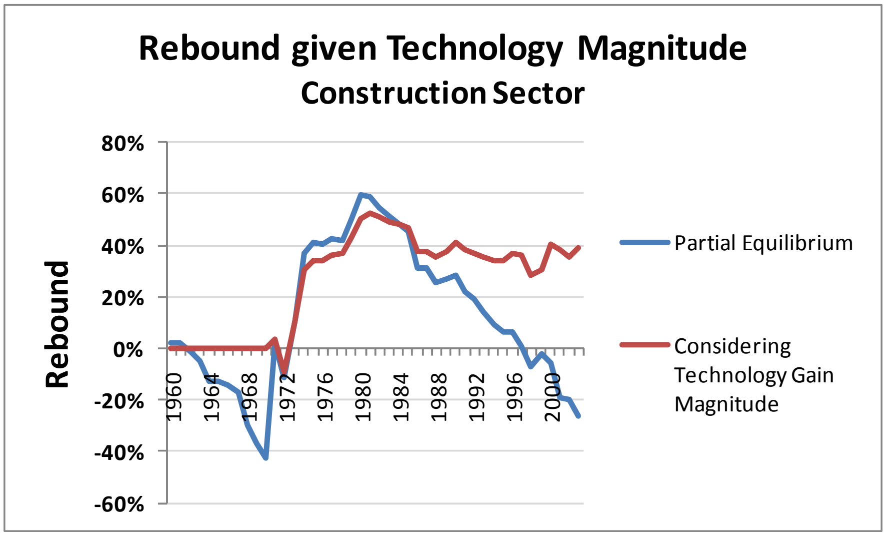

Figure B-2. Actual Rebound Characteristics of Newest Vintage (Energy technology gain only)

This figure shows that, for the Construction sector, the combination of the larger-than-unity technology gain (1.63% per year) and the cumulating output effect is sufficient to offset the decline in inherent rebound propensity.[48] Note that this can work either way—a smaller-than-unity technology gain in a sector will show the inherent (or "partial equilibrium") rebound to be above the actual rebound characteristics of the newest vintage.

Consideration of Output Demand Elasticities

The final element contributing to the dynamic evolution of rebound is the effect of energy technology gains on the consumption of output for any sector. Energy technology gains enter the unit cost function, reducing output price, which will result in increased consumption of output.

The magnitude of this effect will in reality depend in a complex way on how consumers' utilities for this good/service relate to their utilities for other goods/services whose price will likewise depend on the energy technology gains in those sectors. For a general-equilibrium-consistent comparison, as described in the main text, the figure below shows the result for a Cobb-Douglas consumer utility function:

[48] Note that in the "considering technology gain magnitude" case efficiency gains do not begin until 1970, the reference year chosen for the Ryan-Wales concavity procedure.

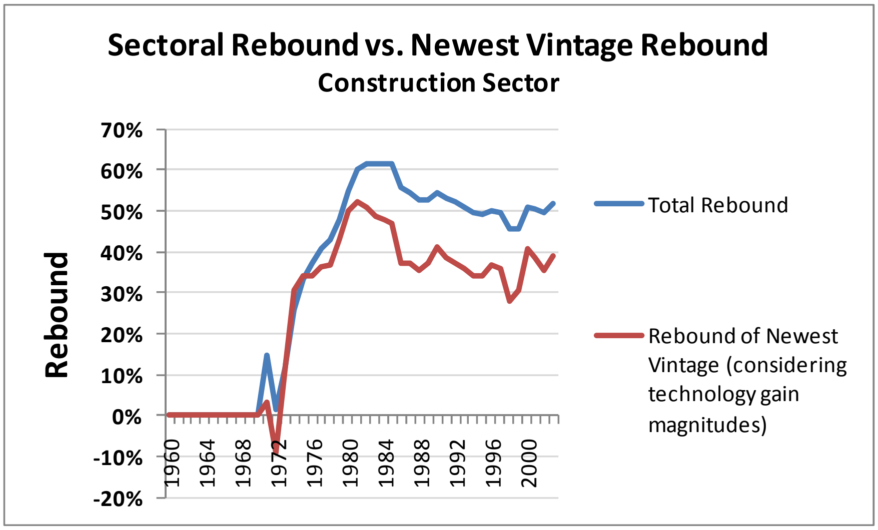

Figure B-3. Sectoral Rebound vs. Actual Rebound Characteristics of Newest Vintage

Output price reductions act upon all output and factor uses across all vintages, not just the newest. In the case of the Construction sector, total rebound of sector-wide energy consumption is increased by introduction of the output demand elasticity effect, even while remaining heavily influenced by the rebound characteristics of the newest vintage.

Further, the output effects embodied in the capacities of the newest vintages will cumulate as they work their way through the sector over time.

Appendix C: Methods Combinations

As described in the Methodology section of the main article and in Appendix A, approximately a hundred combinations of methods are tested for each sector to find the combination providing the best explanatory power and conformance with statistical, concavity and forecast deviation metrics. This Appendix summarizes the methods combinations chosen.

First, there are different combinations of methods for setting up the data for econometric measurement. The table below summarizes across the 30 sectors the chosen combinations.

	Econometric Methods Used							
	Capital Formation Method			Utilization Profile			p_K Adjustment	
	Shephard's Lemma	Revenue Reinvestment	Cash Flow Reinvestment	Uniform Across Vintages	Negative Cash Flow Vintages Permanently Removed from Service	Negative Cash Flow Vintages Temporarily Removed from Service	Yes	No
Number of Sectors	**29**	**1**	**0**	**30**	**0**	**0**	**26**	**4**

Table C-1. Summary of Econometric Method Combinations Chosen

As shown, the Shephard's Lemma method almost universally provides the best explanatory power, with the Revenue Reinvestment method working best for only one sector, the Services sector. Disappointingly, and surprisingly, cash flow-based methods for both capital formation and for distributing utilization rates across vintages never seem to work well, but it is perhaps useful to have considered them. The utilization-based adjustment to capital price almost always provides the best performance. In the four sectors where this adjustment does not appear to work as well, it is possible that data problems with utilization rates are to blame. There is not always an exact mapping of sector definitions from the utilization rate data to those of the Jorgenson et al. data. Further, in two of these four sectors, Stone, Glass & Clay and Services, no utilization rate data are available and industry-wide data were used. All this strongly suggests such an adjustment should be invoked in an analysis like this.

Not shown, as they are not easily summarized, are the different choices of reference year for applying the Ryan & Wales concavity procedure. Also, different domains of the econometric measurement must be tested to account for the limitation that technology gains are assumed to be uniform over time. However, time horizons encompassing no less than 20 years of the 40-year data set, judiciously chosen, deliver good results. This suggests the assumption of uniform technology gains may not be too worrisome an assumption.

The best choice of econometric setup method is not revealed until the various rebound method combinations are examined. Accordingly, multiple econometric method combinations are tested against multiple rebound method combinations. The results are summarized below:

	Rebound Methods Used								
	Capital Formation Method	Utilization Profile	p_K Adjustment	Delta Y Method					
				Accounting Method	Ohti's Theorem Method				
					Price Elasticity Form	Delta X Form (Prices)	Delta X Form (Shares)	Total
Number of Sectors	***To match Econometric Method***	***To match Econometric Method***	***To match Econometric Method***	**6**	**11**	**5**	**8**	**24**

Table C-2. Summary of Rebound Method Combinations Chosen

To ensure consistency, in each sector the capital formation, utilization and capital price adjustment methods chosen are made to match those used in the particular econometric method combination being tested, despite the fact that the toolkit can readily "mix and match" between them if one chooses.

To create the output capacity of the newest vintage, the method involving the extension to Ohti's Theorem most frequently provides the best performance, although the Accounting method works best 20% of the time. Within the Ohti's Theorem method the Price Elasticity form most often performs best, although the Delta X forms not infrequently do better. This dominance of the Ohti's Theorem method, relying as it does on theoretical duality principles arising from an assumption of perfectly competitive profit-maximizing firms, may suggest that these assumptions are quite descriptive of actual firm behavior.