Tilt Detection Using Accelerometer and Barometric Measurements

Can this be used for reliable off-axis detection?

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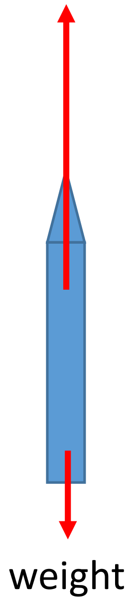
Topics

- Accelerometer altimeter altitude calculation math for a vertical flight
- Accelerometer altitude calculation for an off-vertical flight
- Off-vertical angle versus accelerometer altitude error
- Is this practical with real world sensors and measurement uncertainty?
- Sources of measurement error
- Measurement system requirements to resolve off-vertical tilt angle
- Monte-Carlo analysis of expected error distributions and correct measurement probabilities.
- Use Case: Staging ignition decision
- Summary

Math of altimeter acceleration altitude calculation

Assume a flight where (Thrust-Drag)/Weight = 7 for 4 seconds
Determine the altitude at motor burnout (4 seconds)

Thrust = 7 x weight



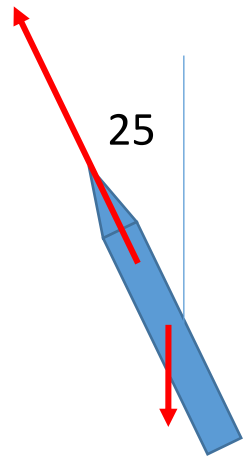
F=ma; a = F/m
Acceleration a = (7mg – mg)/m = 6g

(But during flight the accelerometer will read 7g's because the accelerometer cannot measure the force due to gravity)

Altitude = ½ * a * t^2
Altitude = ½ *6*32.2*16 = 1545.6 ft

To calculate altitude from the altimeter reading you need to subtract 1g from the reading which assumes a purely vertical flight

Now lets assume that the rocket flies 25 deg off from vertical



Acceleration a = [7mg – cos(25)mg]/m = 6.1g

The accelerometer will still read 7g.
The actual acceleration relative to the ground will be 6.1g.

Distance vertically = (½ * a * t^2)*cos(25)
Distance vertically = (½ *6.1*32.2*16)*cos(25) = 1424.1 ft

Acceleration based altitude will be
Accel Alt = ½*6*32.2*16 = 1545
Because the accelerometer will still read the same as the vertical case.

So there will be an ~8.6% difference between the true altitude and the accel based altitude

The same motor will yield more acceleration because its working against less gravity at 25 degrees.

However the accelerometer will still read the identical value as the vertical flight

Accelerometer altitude error versus off-vertical angle

So in theory we can deduce the off axis angle from difference between the acceleration based altitude and the true altitude.

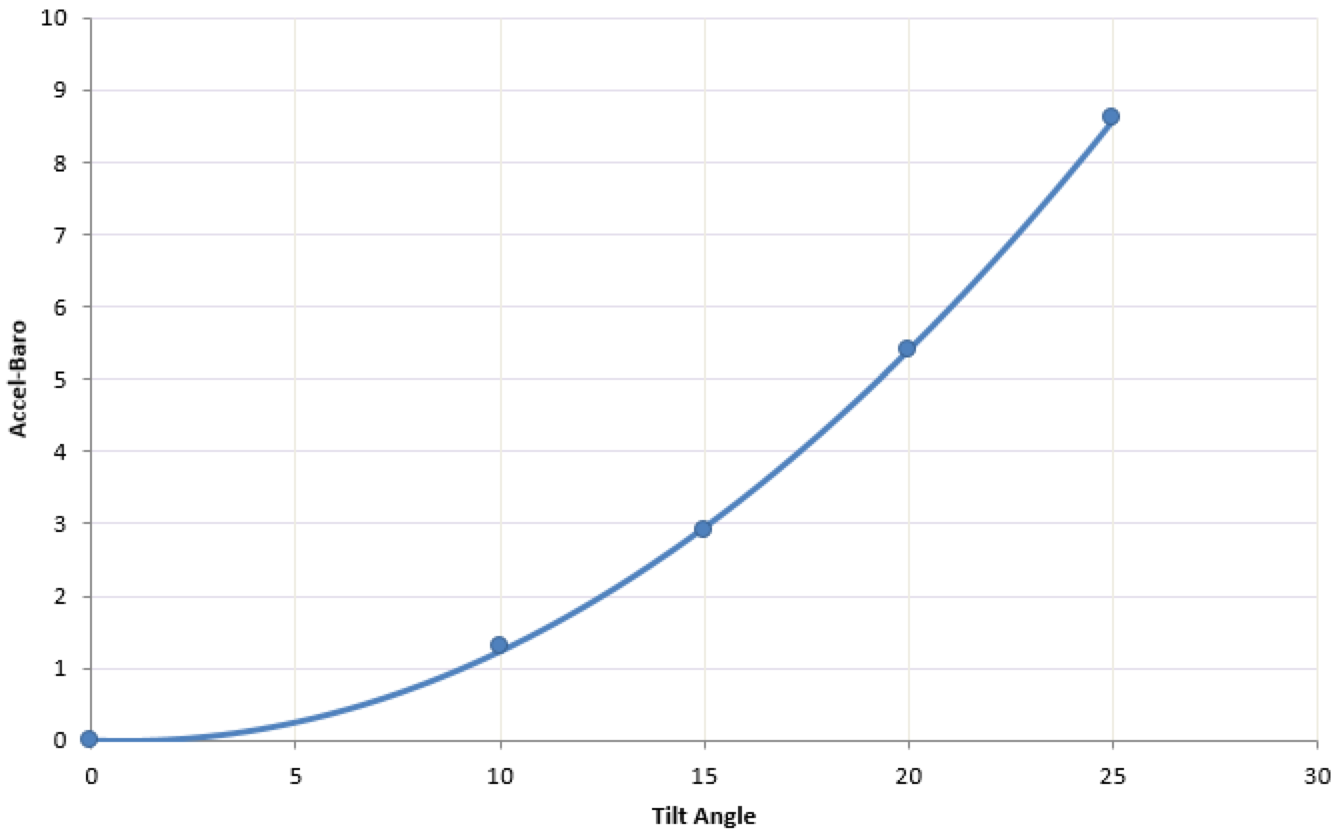
Angle (Tilt) degrees	0	10	15	20	25
Motor Thrust (mg)	7	7	7	7	7
mg component along thrust axis	1.000	0.985	0.966	0.940	0.906
Net thrust (mg)	6.000	6.015	6.034	6.060	6.094
Acceleration along thrust axis (ft/s2)	193.2	193.7	194.3	195.1	196.2
Distance traveled along thrust axis (ft)	1545.6	1549.5	1554.4	1561.1	1569.7
True vertical altitude	1545.6	1526.0	1501.4	1467.0	1422.7
Accel alt versus true altitude error	0.00	19.63	44.19	78.61	122.94
Accel alt error %	0.0%	1.3%	2.9%	5.4%	8.6%

Measurement System Precision Requirements

The transfer function of off-vertical tilt angle to accel-baro percentage discrepancy can be effectively modeled with quadratic curve fit of the data.

Scatter plot for Tilt Angle vs Accel-Baro

Data source: 'Sheet1'C6:D10



Enter the X value, and Quantum XL will calculate the expected Y value and display it on the graph. Note: Quantum XL must be running in order to update the chart's Min and Max values.

Y Prediction		
X: Tilt_Angle	**(enter)**	← Enter X Value Here
Y: AccelBaro		← Predicted Y

Regression analysis -Polynomial fit, 2nd order

AccelBaro = **0.014672** * Tilt_Angle ^{2} - **0.023814** * Tilt_Angle + **0.012077**

	Coeff.	t-Statistic	p-Value	Tolerance
Const	0.012077	0.20793	0.855	
Tilt_Angle	-0.023814	-2.3691	0.141	**0.092965**
Tilt_Angle^2	0.014672	37.692	0.001	**0.092965**

Count	R^2	Adj R^2	F	Std Error
5	0.99985	0.9997	6,756.2	0.058955

Accelerometer altitude error versus off-vertical angle

Is this practical with real sensors?

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True value unknowable with real sensors

The potential problem is the error or uncertainty of the acceleration and barometric measurements may be greater than the difference we need to resolve to infer tilt

Sources of error – Sensors

Accelerometer errors - Spec samples from ST AIS1120SX datasheet

Error Source	Spec	Note
Sensitivity	+/- 5%	Average sensitivity error across the range
Non-linearity	+/- 2% FS	Maximum deviation from linearity
Package alignment	-5%	Alignment of MEMs sensor to the IC package. Would only cause an under measurement
Altimeter alignment	2 deg	Axis mounting error in the rocket – would only cause under measurement

Barometric errors - Spec sample from typical MeasSpec/Bosch pressure sensor

Error Source	Spec	Note
Relative Pressure	+/- 5 feet	Differential pressure error converted to height
Pressure reading lag	33 ms	There will be a time lag in the pressure reading due to the filtering applied in the sensor and the venting of the altimeter bay. This error will be worse at high speeds and maximum at motor burnout. This will cause and under measurement.
SAM error	+/- 5%	This error is due to atmospheric conditions not matching the Standard Atmospheric model or the model that is used to convert pressure to altitude.

Other errors that maybe significant but not considered in this analysis

- Digital quantization errors
- Digital timing and integration clock error
- Accumulated error of numerical integration method chosen
- Av-bay pressure anomalies due to:
    - Poor venting
    - Turbulence due to airframe discontinuities
    - Mach transitions

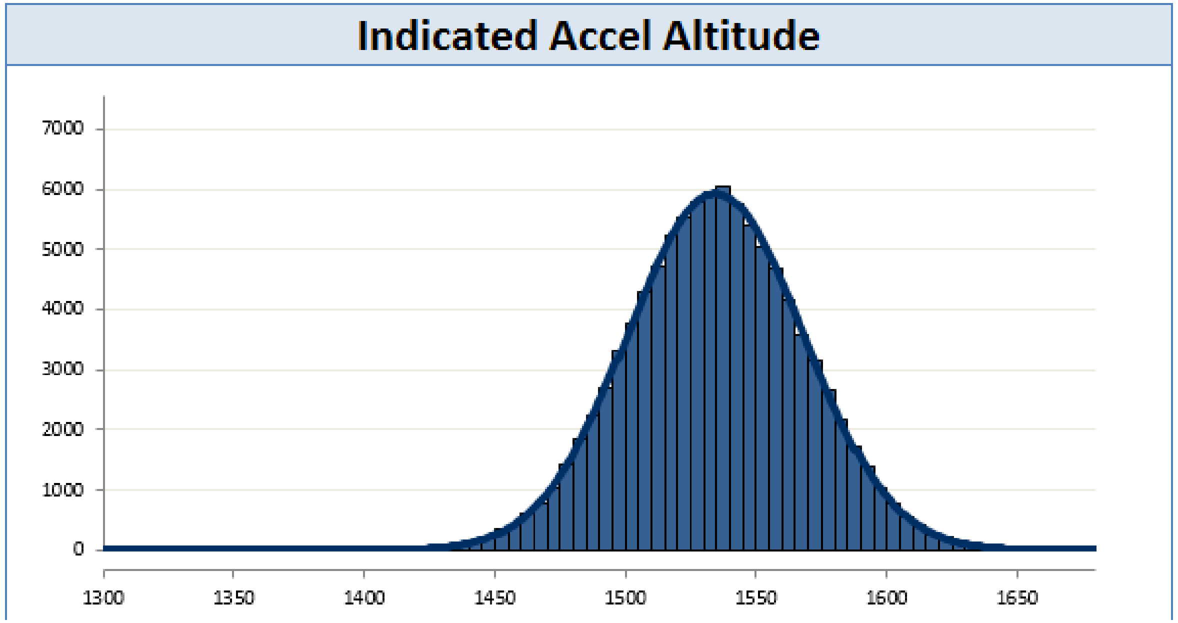
Monte-Carlo analysis to determine the normal expected error between barometric and accelerometer derived altitudes.

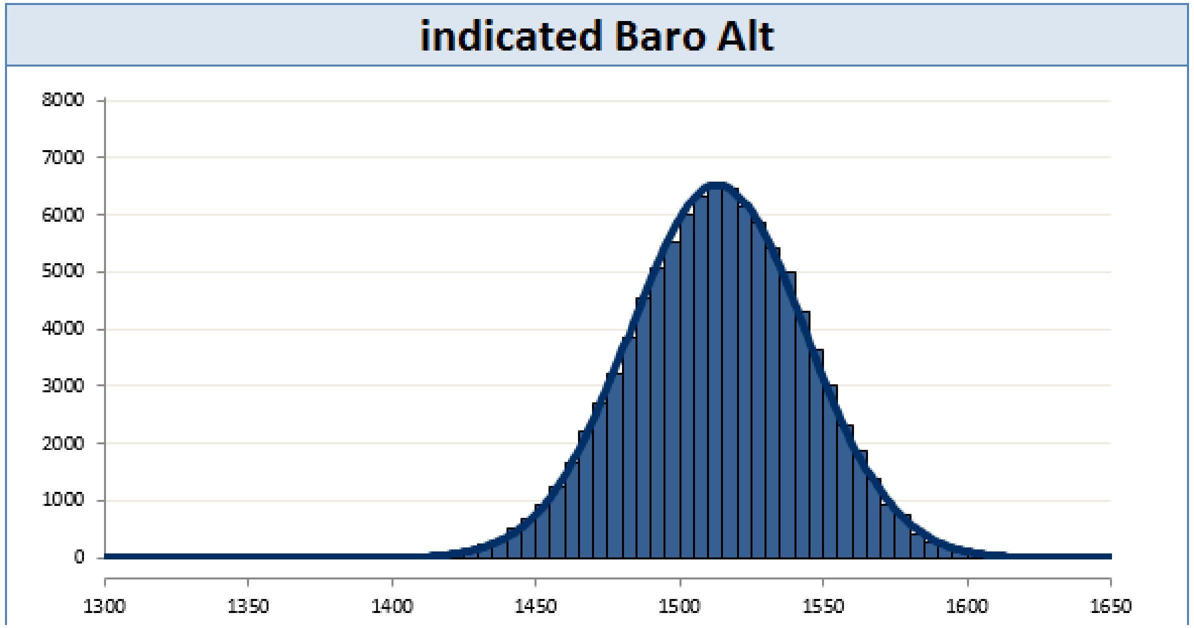
To calculate the error of the measurement system we are going to simulate 100,000 pure vertical flights with a 7 T/W motor and simulate what the baro altitude and accelerometer altitude would be returned at motor burnout which occurs a 1545 ft.

For each simulation a random error for each error will be selected in the range that is allowed in the error sour table on slide 4. After 100,000 such simulations we will get a distribution of the reported accel and baro altitudes and the percentage difference between these 2 quantities. We can compare this distribution to the precision required to determine off axis tilt angle.

Monte-Carlo analysis to determine the normal expected error between barometric and accelerometer derived altitudes.

Here are the results of the 100,000 simulated flights…….





As can be seen in the distribution plots above, even on straight up flights significant differences between accelerometer and barometric altitude can be expected.

Use Case: Detect tilt angle and decide whether to ignite sustainer motor on a 2-stage flight

The common use case for tilt detection is to inhibit 2nd stage ignition for an off vertical flight.

An ideal system would command ignition if the tilt angle is less than a preset limit and not command an ignition if the tilt angle exceeds this limit.

There are 2 error states possible:

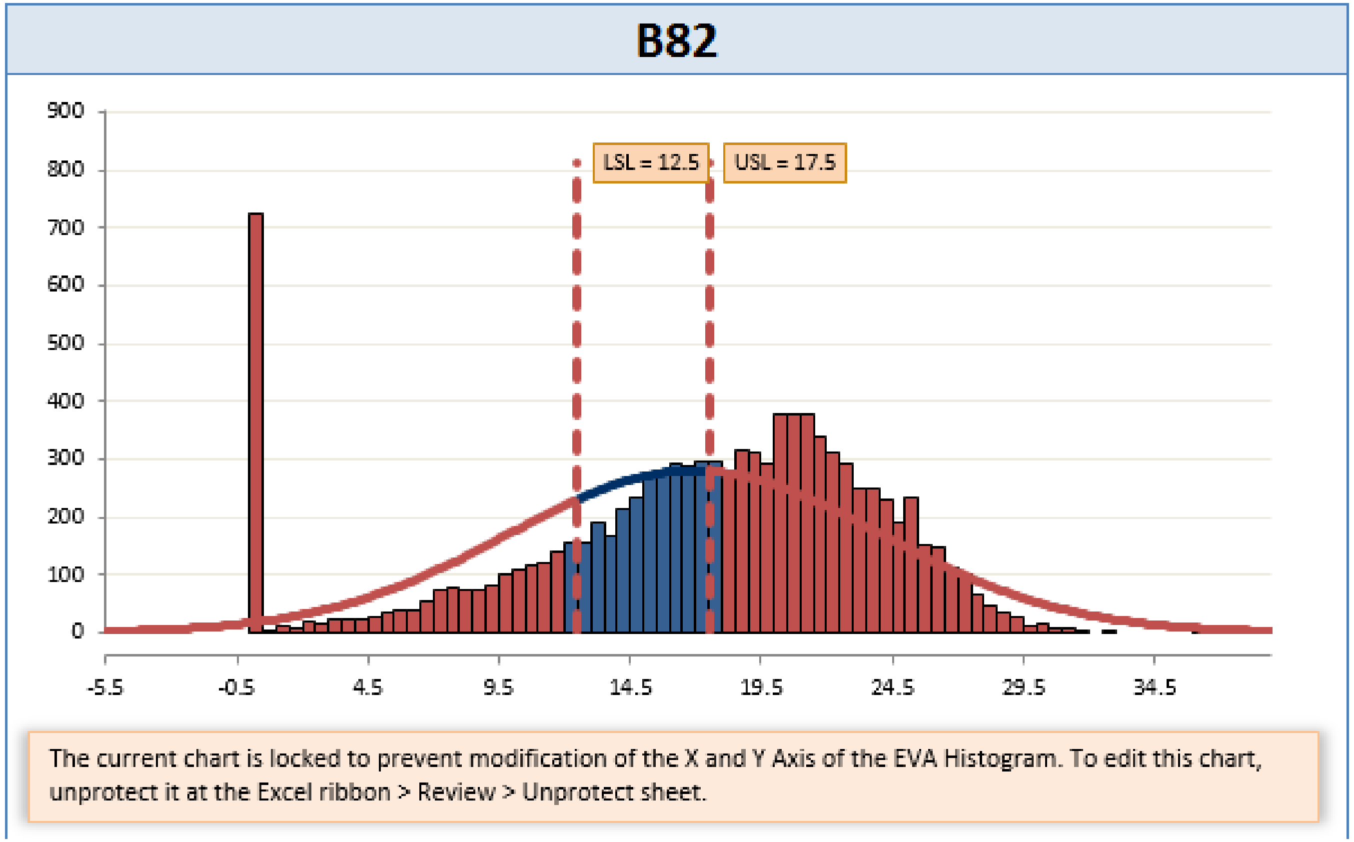
1. False negative: The system FAILS to inhibit ignition when the tilt angle exceeds the limit.

2. False positive: The system FAILS to command ignition even though the tilt angle is less than the allowable limit.

Use Monte-Carlo simulation to determine the probability of each error type.

Simulate a 15 degree off vertical flight and determine the distribution of possible measurements thereof.

Assume a +/- 2.5 deg allowable error (IE. Want an ignition at less than 12.5 degrees and want to inhibit ignition greater than 17.5 degrees.



This is the probability distribution of “tilt” measurements for a 15 deg flight with random combination of possible systemic errors.

Red bars are ERRORS, blue bars correct measurements within our tolerance allowance.

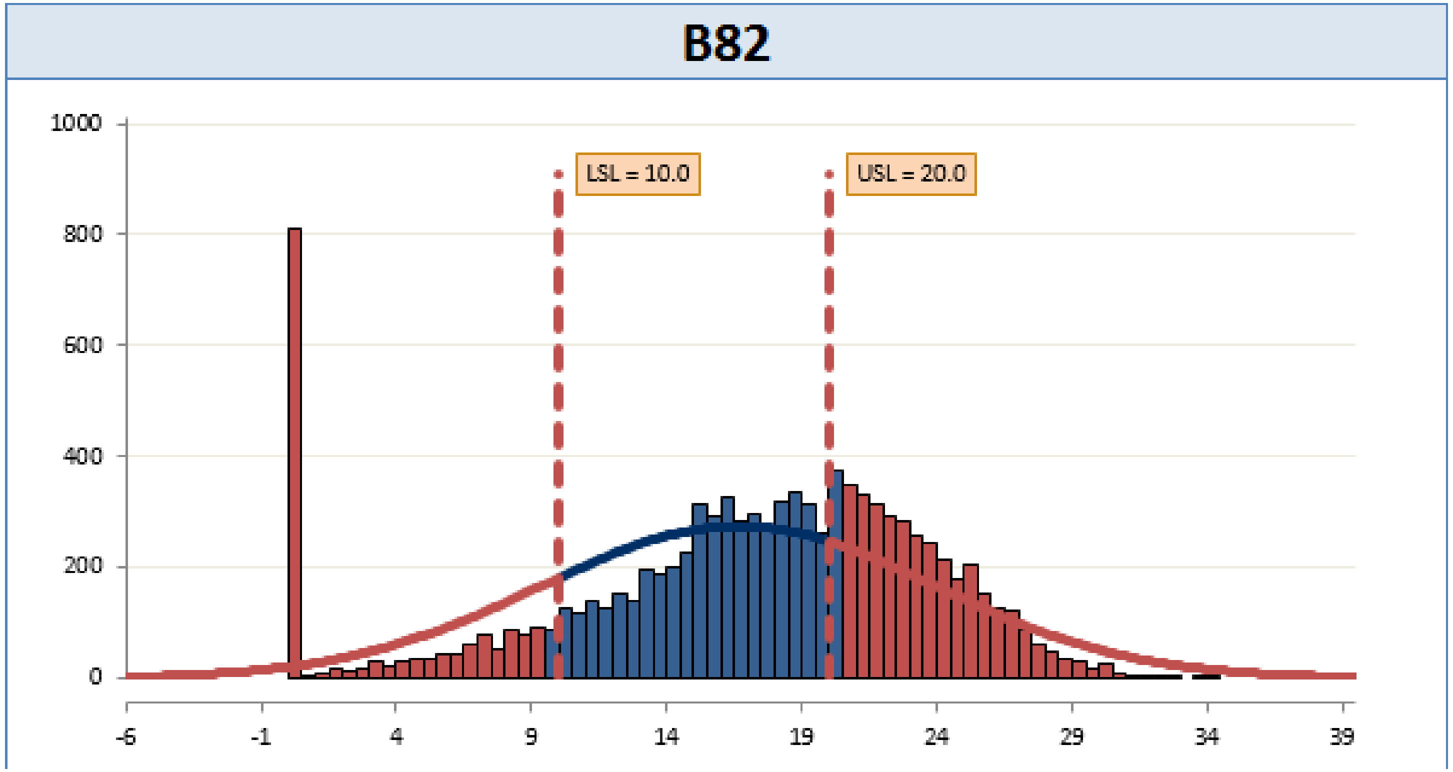
False negative Prob: ~25%
False positive Prob: ~55
Prob of correct measurements: **20%**

MEASUREMENT SYSTEM NOT CAPABLE @ +/- 2.5 deg

Is system capable at +/- 5 deg error band?

Simulate a 15 degree off vertical flight and determine the distribution of possible measurements thereof.

Assume a +/- 5 deg allowable error (IE. Want an ignition at less than 12.5 degrees and want to inhibit ignition greater than 17.5 degrees.



This is the probability distribution of "tilt" measurements for a 15 deg flight with random combination of possible systemic errors.

Red bars are ERRORS, blue bars correct measurements within our tolerance allowance.

Prob of correct measurements: **40%**

MEASUREMENT SYSTEM NOT CAPABLE @ +/- 5 deg

Summary and Conclusions

- Accelerometer versus Barometric altitude difference method does not have sufficient measurement capability to be a *useful* staging inhibition system for moderate (10 to 20 deg) off vertical flights. False positives and negatives can happen 50% of the time.

- The largest contributor to system measurement error is the systemic error of barometric altitude lag at velocity and SAM assumption error.

- This method can be used for detection of very large (>45 deg) off axis flights.