A Macroeconomic Model with Financially Constrained Producers and Intermediaries *

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Abstract

We propose a model that can simultaneously capture the sharp and persistent drop in macro-economic aggregates and the sharp change in credit spreads observed in the U.S. during the Great Recession. We use the model to evaluate the quantitative effects of macro-prudential policy. The model features borrower-entrepreneurs who produce output financed with long-term debt issued by financial intermediaries and their own equity. Intermediaries fund these loans combining deposits and their own equity. Savers provide funding to banks and to the government. Both entrepreneurs and intermediaries make optimal default decisions. The government issues debt to finance budget deficits and to pay for bank bailouts. Intermediaries are subject to a regulatory capital constraint. Financial recessions, triggered by low aggregate and dispersed idiosyncratic productivity shocks result in financial crises with elevated loan defaults and occasional intermediary insolvencies. Output, balance sheet, and price reactions are substantially more severe and persistent than in non-financial recession. Policies that limit intermediary leverage redistribute wealth from producers to intermediaries and savers. The benefits of lower intermediary leverage for financial and macro-economic stability are offset by the costs from more constrained firms who produce less output.

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1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have caused economists to revisit the role of the financial sector in models of the macro economy. Building on early work that emphasized the importance of endogenous developments in credit markets in amplifying business cycle shocks,[1] a second generation of models has added nonlinear dynamics and a richer financial sector.[2] While a lot of progress has been made in understanding how financial intermediaries affect asset prices and macroeconomic performance, an important remaining challenge is to deliver a quantitatively successful model that can capture the dynamics of financial intermediary capital, asset prices, and the real economy during normal times and credit crises. Such a model requires a government, so that possible crisis responses can be studied, and explicit and implicit government guarantees to the financial sector can be incorporated. Indeed, Central Banks are in search of a model of the financial sector that can be integrated into their existing quantitative macro models. Our paper aims to make progress on this important agenda. It provides a calibrated model that matches key features of the U.S. macroeconomy and asset prices. In addition, it makes three methodological contributions.

First, we separate out the role of producers and banks. The existing literature, as exemplified by the seminal Brunnermeier and Sannikov (2014) paper, combines the roles of financial intermediaries and producers ("experts"). This setup assumes frictionless interaction between banks and borrowers and focuses on the interaction between experts and saving households. It implicitly assumes that financial intermediaries hold equity claims in productive firms. In reality, financial intermediaries make corporate loans and hold corporate bonds which are debt

[1] E.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and Gertler and Karadi (2011).

[2] E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016).

claims.[3] These debt contracts are subject to default risk of the borrowers. Our model has three groups of agents, each with their own balance sheet: savers who lend to intermediaries, entrepreneurs who own the production technology and borrow from intermediaries, and bankers who intermediate between the depositors and entrepreneurs. Intermediaries perform the traditional role of maturity transformation and bear most of the credit risk in the economy. They help to optimally allocate risk across the various agents in the economy. Costly firm bankruptcies endogenously limit the debt capacity of entrepreneurs. In order to discipline banks, we model a Basel-style regulatory capital requirement that limits banks' liabilities at a fraction of their risk-weighted assets. The minimum regulatory capital that banks must hold is a key macroprudential policy parameter.

Our second contribution is to introduce the possibility of default for financial intermediaries. The existing literature is usually cast in continuous time. As the financial sector approaches insolvency, intermediaries reduce risk and prices adjust so that they never go bankrupt. In discrete time, the language of quantitative macroeconomics, the possibility of default of intermediaries cannot be avoided. Far from a technical detail, bank insolvency is an important reality that keeps policy makers up at night. As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2014) make clear, financial intermediaries frequently become insolvent. When they do, their creditors (mostly depositors) are bailed out by the government. In our model we assume that intermediaries have limited liability and choose to default optimally. When the market value of their assets falls below that of liabilities, the government steps in, liquidates the assets and makes whole their creditors. The banking sector starts afresh the next period with zero wealth. The expectation of a bailout affects banks' risk taking incentives (e.g., Farhi and Tirole (2012)). By allowing for the possibility of bank insolvencies, our model can help explain how a corporate default wave can trigger financial fragility. Vice versa, weak financial balance sheets reduce firms' ability to borrow, invest, and grow.

The third methodological contribution is to endogenize the risk-free interest rate on safe debt. Most models in the intermediary-based macro and asset pricing literature keep the interest rate on safe assets (deposits or government debt) constant, sometimes by virtue of an assumption of risk neutrality of the savers. Once savers are risk averse, a natural assumption

[3] It is well understood that debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See for example Dang, Gorton, and Holmstrom (2015).

given that they invest in guaranteed deposits, the dynamics of the model change substantially. In a crisis, intermediaries contract the size of their balance sheet, thereby reducing the supply of safe debt in the economy. Simultaneously, risk averse depositors with strong precautionary savings motives increase their demand for safe assets. As a result, the equilibrium price of safe debt increases substantially. Real interest rates fall sharply. The low cost of debt allows the intermediaries to recapitalize quickly, dampening the effect of the crisis. Put differently, the endogenous price response of safe debt short-circuits the amplification mechanism that arises in a balance sheet recession in partial equilibrium models that hold the interest rate fixed.[4] A partial solution lies in carefully modeling the government side of the model. With counter-cyclical spending and procyclical tax revenues, the government deficit is counter-cyclical. This expands the supply of safe debt in bad times, offsetting the contraction in the supply by the intermediation sector. While rates may still fall in crises, the decline is not as large as it would be without the government sector, and restores the amplification of the balance sheet recession models. Importantly, because the risk averse saver must absorb more debt in bad times, she must reduce spending in high marginal utility states. The ex-ante precautionary savings effect this triggers reduces the unconditional mean interest rate in the economy. While automatic stabilizers in fiscal policy may still be desirable for aggregate welfare, a new insight is that they slow down the recapitalization of banks in a crisis through their general equilibrium effect on the real interest rate.

What results is a rich and quantitatively relevant framework of the interaction between four balance sheets: those of borrower-entrepreneurs, financial intermediaries, saving households, and the government, featuring occasionally binding borrowing constraints for both borrower-entrepreneurs and for intermediaries, and bankruptcy of both borrowers and intermediaries. The model generates amplification whereby aggregate shocks not only directly affect production and investment, but also affect the financial and non-financial sectors' leverage. Tighter financial constraints on banks reduce the availability of credit to firms which hurts investment and output, beyond the effects familiar from standard accelerator models.

[4] One might argue that there are other investors in the market for safe assets whose demand for safe assets may not rise as much because they are less risk averse (maybe institutional investors), but their demand for safe debt would have to be negatively correlated with that of the risk averse savers to offset the effect. Foreigners' demand for U.S. safe debt also increased dramatically in the global financial crisis, further amplifying domestic demand by savers rather than offsetting it.

Our model quantitatively matches the maturity, default risk, and loss-given default of corporate debt. It generates a large and volatile credit spread, again matching the data. The endogenous price of credit risk dynamics amplify the dynamics in the quantity of credit risk. Intermediary wealth fluctuations are behind this resolution of the credit spread puzzle (e.g., Chen (2010)). We use the model to study the differences between regular non-financial recessions and financial recessions, which are recessions that coincide with credit crisis.

Our second main exercise is to investigate the quantitative effects of macro-prudential policies for financial stability, economic growth, economic stability, fiscal stability, and economy-wide welfare. Our model belongs to the class of models where incomplete markets and borrowing constraints create room for macro-prudential policy intervention.[5] We find that while macro-prudential policies improve financial stability and reduce macroeconomic volatility, they also shrink the size of the economy. On net, a reduction in maximum bank leverage has large redistributional consequences shifting wealth from borrowers and savers towards intermediaries. It has modest negative effects on aggregate welfare. Our model offers a quantitative answer to this important policy question.

Our paper provides a state-of-the-art solution technique. The model has two exogenous and persistent sources of aggregate risk. Standard TFP shocks hit the production function. In addition, shocks to the cross-sectional dispersion of idiosyncratic firm productivity govern credit risk. The model also has five endogenous aggregate state variables: the capital stock, corporate debt stock, intermediary net worth, household wealth, and the government debt stock. To solve this complex problem, we provide a nonlinear global solution method, called policy time iteration, which is a variant of the parameterized expectations approach. Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of nonlinear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities.[6]

[5]Other models in this class are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2013), Bianchi and Mendoza (2015), and Guerrieri and Lorenzoni (2015). Farhi and Werning (2016) study macroprudential policy in a model with demand externalities.

[6]One output of this research project will be a set of computer code which will be made publicly available. Discussions with the research department at three different Central Banks indicate that there is a demand for this type of output. Our method improves on existing methods which compute two non-stochastic steady states: one steady state when the constraint never binds and one where it always binds, and then linearizes the solution around both of these states. In this approach, agents inside the model do not take into account the fact that borrowing constraints may become binding in the future due to future shock realizations. As a

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations and some details on the calibration are relegated to the appendix.

2 The Model

2.1 Preferences, Technology, Timing

Preferences The model features a government and three groups of households: borrower-entrepreneurs (denoted by superscript B), intermediaries (denoted by superscript I) and savers (denoted by S). Savers are more patient than borrower-entrepreneurs and intermediaries, implying for the discount factors that $\beta_B = \beta_I < \beta_S$. For the coefficient of relative risk aversion we assume that $\sigma_I = \sigma_B \leq \sigma_S$. All agents have Epstein-Zin preferences over utility streams $\{u_t^j\}_{t=0}^{\infty}$ with intertemporal elasticity of substitution ν_j.

$$U_t^j = \left\{ (1-\beta) \left(u_t^j \right)^{1-1/\nu_j} + \beta_j \left(\mathrm{E}_t \left[(U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu_j}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu_j}}, \tag{1}$$

for $j = B, I, S$. Savers have a higher intertemporal elasticity of substitution: $\nu_S > \nu_B = \nu_I$. Intermediaries and savers only derive utility from consumption of the economy's sole good, such that $u_t^j = C_t^j$, for $j = I, S$. Borrower-entrepreneurs also derive disutility from providing managerial effort m_t, such that $u_t^B = u^B(C_t^B, m_t)$.

Technology Borrower-entrepreneurs own the productive capital stock of the economy and operate its production technology of the form

$$Y_t = m_t \, (K_t)^{(1-\alpha)} (Z_t L_t)^{\alpha}, \tag{2}$$

result, the approach ignores agents' precautionary savings motives related to future switches between "regimes" with and without binding constraints. While the piecewise-linear solution may prove sufficiently accurate in some contexts, it remains an open question whether it offers an appropriate solution to models with substantial risk and higher risk aversion, designed to match not only macroeconomic quantities but also asset prices (risk premia). See Guerrieri and Iacoviello (2015) for a nice discussion on these issues.

where K_t is capital, L_t is labor, Z_t is labor productivity, and m_t is aggregate managerial effort of borrower-entrepreneurs. Section 2.2 and appendix A.1 derive this aggregate production function based on a model of entrepreneurial effort provision with a continuum of entrepreneurs that receive idiosyncratic productivity shocks and have to provide costly managerial effort to run their firms. We assume that productivity Z_t grows at a stochastic rate g_t which follows an AR(1) process.

In addition to the technology for producing consumption goods, borrower-entrepreneurs also have access to a technology that can turn consumption into capital goods.

Borrower-entrepreneurs, intermediaries, and savers are endowed with $\bar{L}^B$, $\bar{L}^I$ and $\bar{L}^S$ units of labor, respectively. We assume that all types of households supply their labor endowment inelastically.

There are two more assets in the economy. One risky long-term bond that borrower-entrepreneurs can issue to intermediaries (corporate loans), and one short-term risk free bond that intermediaries can issue to savers (deposits).

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Aggregate and idiosyncratic productivity shocks for borrower-entrepreneurs are realized. Production occurs.

2. Intermediaries decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown. All agents know its probability distribution, and intermediaries maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[7]

3. Borrower-entrepreneurs with low idiosyncratic productivity realizations default. Intermediaries assume ownership of bankrupt firms.

[7]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)). The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

4. Intermediaries' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of intermediary bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Figure 1: Overview of Balance Sheets of Model Agents

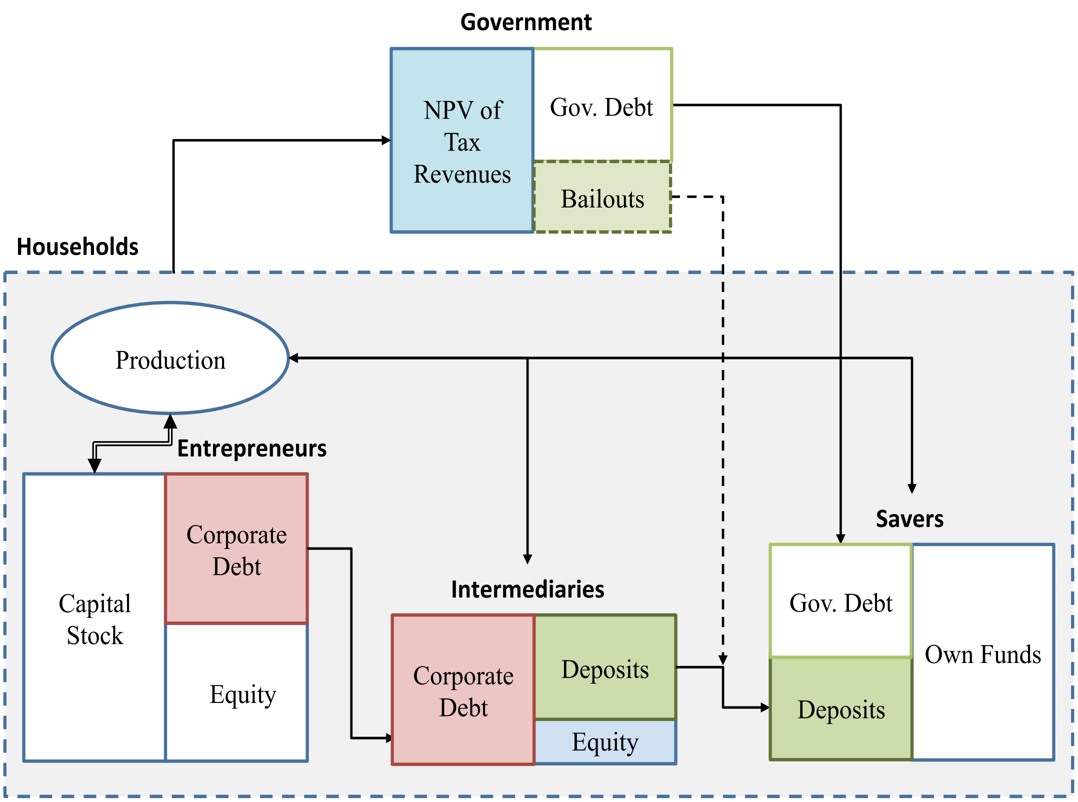


Figure 1 illustrates the balance sheets of the model's agents and their interactions. Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries. We now describe each of the three types of household problems and the government problem in detail.

2.2 Borrower-Entrepreneurs' Problem

There is a unit-mass of identical borrower-entrepreneurs indexed by i. The households form a large collective ("family") that provides partial insurance against idiosyncratic shocks.

Each entrepreneur has access to a technology that creates consumption goods $Y_{i,t}$ from capital $K_{i,t}$ and labor $L_{i,t}$. Output depends on aggregate productivity Z_t and idiosyncratic productivity g_{i_t}:

$$Y_{i,t} = g_{i,t} K_{i,t}^{1-\alpha} (Z_t L_{i,t})^{\alpha}.$$

At the beginning of the period, each entrepreneur receives an idiosyncratic productivity shock $\omega_{i,t} \sim F_{\omega,t}$, distributed independently over time. The individual productivity is a function of the idiosyncratic shock and individual effort e_i, where $0 < \phi < 1$ parameterizes the elasticity of effort with respect to changes in productivity:

$$g_i = g(\omega_i, e_i) = \omega_i^{\phi} e_i^{1-\phi}.$$

Only the individual entrepreneur can observe her idiosyncratic ω_i shock and her effort e_i. However, idiosyncratic output and hence productivity $g_{i,t}$ is observable to everyone. Each entrepreneur has preferences over consumption and effort

$$u(C_i, e_i) = \log(C_i) - \eta e_i, \tag{3}$$

with parameter $\eta > 0$.

While each individual entrepreneur manages her own production, the family of borrower-entrepreneurs manages the allocation of production inputs and consumption. Further, the family collectively issues debt to intermediaries. The debt is long-term, modeled as perpetuity bonds. Bond coupon payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the bond. We introduce a "face value" $F = \frac{\theta}{1-\delta}$, a fixed fraction θ of all repayments for each bond issued. Per definition, interest payments are the remainder $\frac{1-\theta}{1-\delta}$.

At the beginning of the period, the family jointly holds K_t^B units of capital, and has A_t^B bonds outstanding. In addition, producers jointly hire their own labor and the labor of intermediaries and savers, denoted by L_t^j, with $j = B, I, S$. As payment each group receives a competitive wage w_t^j per unit of labor. During production, the labor inputs of the three types are combined into aggregate labor:

$$L_t = (L_t^B)^{1-\gamma_S-\gamma_I}(L_t^S)^{\gamma_S}(L_t^I)^{\gamma_I}.$$

Before idiosyncratic productivity shocks are realized, each producer is given the same amount of capital and labor for production, such that $K_{i,t} = K_t^B$ and $L_{i,t} = L_t$. Further, each producer is responsible for repaying the coupon on an equal share of the total debt, $A_{i,t} = A_t^B$.

The individual profit of producer i is therefore given by

$$\pi_{i,t} = g_{i,t}(K_t^B)^{1-\alpha}(Z_t L_t)^{\alpha} - \sum_j w_t^j L_t^j - A_t^B. \tag{4}$$

After production, each producer who achieves a positive profit, $\pi_{i,t} > 0$, returns this profit to the family. Further, capital depreciates during production by fraction δ_K, and individual members with positive profit return the depreciated capital after production. Producers with *negative* profit default on the share of debt they were allocated. The debt is erased, and the intermediary takes ownership of the bankrupt firm, including its share of the capital stock. The intermediary liquidates the bankrupt firms' capital, seizes their output, and pays their wage bill. The remaining funds are the intermediary's recovery value.

In return for production, each family member receives consumption goods $C_{i,t}$. The family conditions the amount of consumption allocated to each member on observed productivity $g_{i,t}$, by specifying the increasing consumption schedule $C_t(g_{i,t})$.

In appendix A.1 we show that under the following two assumptions:

1. idiosyncratic productivity shocks $\omega_{i,t}$ are distributed Normal$(\mu_\omega, \sigma_{t,\omega}^2)$,
2. the contract between borrower-entrepreneur family and individual entrepreneurs is given by the consumption schedule $C_{i,t} = C_{t,0}C_{t,1}^{g_{i,t}}$, for contract parameters $C_{t,0}$, $C_{t,1} > 1$,

there exists a representative borrower-entrepreneur that has preferences over aggregate borrower consumption C_t^B and effort m_t

$$u_t^B(C_t^B, m_t) = C_t^B \exp\left[-\eta\mu_\omega\left(\frac{m_t}{\mu_\omega}\right)^{\frac{1}{1-\phi}} - 0.5\left(\sigma_{t,\omega}\left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}}\frac{\eta}{1-\phi}\right)^2\right], \tag{5}$$

with aggregate output given by (2). An increase in effort m_t naturally reduces borrower utility. An increase in the cross-sectional dispersion of productivity $\sigma_{t,\omega}$, which we refer to as an "uncertainty shock," reduces borrower utility as well.

We further show that there exists a cutoff productivity shock

$$\omega_t^* = \frac{\sum_{j=B,I,S} w_t^j L_t^j + A_t^B}{m_t/\mu_\omega (K_t^B)^{1-\alpha}(Z_t L_t)^\alpha}, \tag{6}$$

such that all entrepreneurs receiving productivity shocks below this cutoff default on their debt.

Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)$ to be the average productivity of the firms that do not default:

$$Z_A(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*], \tag{7}$$

$$Z_K(\omega_t^*) = \frac{m_t}{\mu_\omega} \Pr[\omega_{i,t} \geq \omega_t^*]\, \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \geq \omega_t^*]. \tag{8}$$

After making a coupon payment of 1 per unit of remaining outstanding debt, the amount of outstanding debt declines to $\delta Z_A\left(\omega_t^*\right) A_t^B$.

The profit of the producers' business is subject to a tax with rate τ_Π^B. The profit for tax purposes is defined as sales revenue net of labor expenses, and capital depreciation and interest payments of non-bankrupt producers[8]

$$\Pi_t^{B,\tau} = Z_K(\omega_t^*)(K_t^B)^{1-\alpha}(Z_t L_t)^\alpha - Z_A(\omega_t^*)\left(\sum_j w_t^j L_t^j + \delta_K p_t K_t^B + (1-\theta)A_t^B\right).$$

The fact that interest expenditure $Z_A(\omega_t^*)(1-\theta)A_t^B$ and capital depreciation $\delta_K p_t K_t^B$ are deducted from taxable profit creates a "tax shield" and hence a preference for debt funding.

In addition to producing consumption goods, producers jointly create capital goods from consumption goods. In order to create X_t new capital units, the required input of consumption goods is

$$X_t + \Psi(X_t/K_t^B)K_t^B, \tag{9}$$

[8] Aggregate producer profit is the integral over the idiosyncratic profit (4) of non-defaulting producers, net of capital depreciation expenses and adding back principal payments θA_t^B which are not tax deductible.

with $\Psi''() > 0$, $\Psi(\mu_G + \delta_K) = 0$, and $\Psi'(\mu_G + \delta_K) = 0$.

The borrower-entrepreneur family's problem is to choose consumption C_t^B, managerial effort m_t, capital for next period K_{t+1}^B, new debt A_{t+1}^B, investment X_t and labor inputs L_t^j to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + Z_A(\omega_t^*)A_t^B(1+\delta q_t^m) + p_t K_{t+1}^B + Z_A(\omega_t^*)\sum_{j=B,I,S} w_t^j L_t^j + \tau_\Pi^B \Pi_t^{B,\tau}$$
$$\leq Z_K(\omega_t^*)(K_t^B)^{1-\alpha}(Z_t L_t)^\alpha + (1-\tau_t^B)w_t^B \bar{L}^B + G_t^{T,B} + p_t(X_t + Z_A(\omega_t^*)(1-\delta_K)K_t^B) + q_t^m A_{t+1}^B, \tag{10}$$

and a leverage constraint:

$$F A_{t+1}^B \leq \Phi p_t (1-\delta_K) Z_A(\omega_t^*) K_t^B. \tag{11}$$

The borrower household uses output, after-tax labor income, net transfer income from the government ($G_t^{T,B}$), sales of old (K_t^B) and newly produced (X_t) capital units, and new debt raised, to pay for consumption, investment including adjustment costs, debt service, new capital purchases, wage and profit tax payments. New debt raised is $q_t^m A_{t+1}^B$, where q_t^m is the price of one bond in terms of the consumption good.

The borrowing constraint in (11) caps the face value of debt at the end of the period, FA_{t+1}^B, to a fraction of the market value of the available capital units after default and depreciation, $p_t(1-\delta_K)Z_A(\omega_t^*)K_t^B$, where Φ is the maximum leverage ratio. With such a constraint, declines in capital prices (in bad times) tighten borrowing constraints. The constraint (11) imposes a hard upper bound on borrower leverage. In addition, costly defaults of individual borrowers who received bad idiosyncratic shocks, endogenously limit the optimal leverage of borrowers. Borrowers take into account that each marginal unit of debt issued in t increases costly defaults in $t+1$. Therefore, for a high enough maximum leverage ratio Φ, the constraint will never be binding.

2.3 Savers

Savers can invest in one-period risk free bonds (deposits and government debt). They inelastically supply their unit of labor $\bar{L}^S$. Entering with wealth W_t^S, the saver's problem is to choose consumption C_t^S and short-term bonds B_t^S to maximize life-time utility U_t^S in (1), subject to the budget constraint:

$$C_t^S + q_t^f B_t^S \leq (1-\tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + W_t^S \tag{12}$$

and a short-sale constraints on bond holdings:

$$B_t^S \quad \geq 0. \tag{13}$$

The budget constraint (12) shows that saver uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, and purchases of short-term bonds

2.4 Intermediaries

After aggregate and idiosyncratic productivity shocks have been realized, financial intermediaries choose whether or not to declare bankruptcy. Intermediaries who declare bankruptcy have all their assets and liabilities liquidated. They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, intermediaries do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \rightarrow \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Intermediaries choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that intermediaries default for all realizations for which the utility cost is below the threshold.

After the realization of the penalty, intermediaries execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem to be described below. First, while intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^I = \frac{C_t^I}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Intermediaries' portfolio choice consists of loans to borrower-entrepreneurs (A_t^I) and short-term bonds (B_t^I). Loans are modeled as bonds aggregating the debt of the borrowers. The coupon payment on performing loans in the current period is $A_t^I Z_A(\omega_t^*)$. For borrower-entrepreneurs that default and enter into foreclosure, the intermediaries repossess their firms, including this period's output, as collateral. Capital and output of the defaulting firms are worth

$$M_t = (1-\zeta)\left[(1 - Z_A(\omega_t^*))(1 - \delta_K)p_t K_t^B + (m_t - Z_K(\omega_t^*))(K_t^B)^{1-\alpha} L_t^\alpha\right] - (1 - Z_A(\omega_t^*))\sum_j w_t^j L_t^j, \tag{14}$$

where ζ is the fraction of capital value and output destroyed in bankruptcy, a deadweight loss. Thus, the total (performing and defaulting) payoff per bond is $Z_A(\omega_t^*) + M_t/A_t^B$. The price of the bond is q_t^m. In addition, intermediaries can trade in short-term bonds with savers and the government. They are allowed to take a short position in these bonds, using their loans to borrower-entrepreneurs as collateral. They are subject to a leverage constraint:

$$-B_t^I \leq q_t^m \xi A_{t+1}^I. \tag{15}$$

A negative position in the short-term bond is akin to intermediaries issuing deposits. The negative position in the short-term bond must be collateralized by the market value of intermediaries' holdings of long-term loan bonds. The parameter ξ determines how useful loans are as collateral. The constraint (15) is a Basel-style regulatory capital constraint. The parameter ξ is the key macro-prudential policy parameter in the paper.

Denote the wealth of an intermediary that did not go into bankruptcy by:

$$W_t^I = Z_A(\omega_t^*)(1 + \delta q_t^m)A_t^I + M_t + B_{t-1}^I \tag{16}$$

Intermediaries are subject to corporate profit taxes at rate τ_Π^I. Their profit for tax purposes is defined as the net interest income on their loan business:[9]

$$\Pi_t^I = (1 - \theta)Z_A(\omega_t^*)A_t^I + r_t^f B_{t-1}^I.$$

Intermediaries' also receive income for inelastically supplying their labor to borrower-entrepreneurs.

[9] We define the risk free interest rate as the yield on risk free bonds, $r_t^f = 1/q_t^f - 1$.

They further need to pay a deposit insurance fee (κ) to the government that is proportional to the amount of short-term bonds they issue. Their budget constraint is

$$(1 - D(\rho_t))W_t^I + (1 - \tau^I)w_t^I \bar{L}^I + G_t^{T,I} \geq C_t^I + q_t^m A_{t+1}^I + (q_t^f + \mathrm{I}_{\{B_t^I < 0\}}\kappa)B_t^I + \tau_\Pi^I \Pi_t^I. \quad (17)$$

Note that intermediaries only receive wealth W_t^I if they do not declare bankruptcy at the beginning of the period; in case of bankruptcy their wealth is zero.

2.5 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues, T_t, are labor income tax receipts plus deposit insurance fee receipts and corporate profit tax receipts:

$$T_t = \sum_{j=B,I,S} \tau_t^j w_t^j L_t^j + \tau_\Pi^B \Pi_t^B + \tau_\Pi^I \Pi_t^I - \mathrm{I}_{\{B_t^I < 0\}}\kappa B_t^I$$

Government expenditures, G_t are the sum of financial sector bailouts, other exogenous government spending, G_t^o, and transfer spending G_t^T:

$$G_t = G_t^o + G_t^T - D(\rho_t)W_t^I$$

The bailout to the financial sector equals the negative of the financial wealth of intermediaries, W_t^I, in the event of a bankruptcy.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \quad (18)$$

We impose a transversality condition on government debt:

$$\lim_{u\to\infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^S B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^S$ is the SDF of the saver.[10]

Because of its unique ability to tax and repay its debt, the government can spread out the cost of default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = (\tau_t^i, \tau_\Pi^i, \kappa, G_t^o, \phi, \xi)$. The parameters ϕ in equation (11) and ξ in equation (15) can be thought of as macro-prudential policy tools. One could add the parameters that govern the utility cost of bankruptcy of intermediaries to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy.

2.6 Equilibrium

Given a sequence of aggregate productivity shocks $\{Z_t\}$, idiosyncratic productivity shocks $\{\omega_{t,i}\}_{i\in B}$, and utility costs of default shocks ρ_t, and given a government policy Θ_t, a competitive equilibrium is an allocation $\{C_t^B, m_t, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}$ for borrower-entrepreneurs, $\{C_t^S, B_t^S\}$ for savers, $\{C_t^I, A_{t+1}^I, B_t^I\}$ for intermediaries, bankruptcy rule $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrower-entrepreneurs, savers, and intermediaries maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

1. Risk-free bonds: $B_t^G = B_t^S + B_t^I$

2. Loans: $A_{t+1}^B = A_{t+1}^I$

3. Capital: $K_{t+1}^B = (1-\delta_K)K_t^B + X_t$

4. Labor: $L_t^j = \bar{L}^j$ for all $j = B, I, S$

[10]We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between $\underline{b^G}$ and $\overline{b^G}$ by decreasing taxes linearly when the debt-to-GDP threatens to fall below $\underline{b^G}$ and raising taxes linearly when debt-to-GDP threatens to exceed $\overline{b^G}$.

5. Consumption:

$$
\begin{aligned}
Y_t =& (C_t^B + C_t^I + C_t^S) + G_t^o + X_t + K_t^B \Psi(X_t / K_t^B) \\
& + \zeta \left[(1 - Z_A(\omega_t^*))(1 - \delta_K) p_t K_t^B + (m_t - Z_K(\omega_t^*))(K_t^B)^{1-\alpha} L_t^\alpha \right]
\end{aligned}
$$

The last equation states that total output equal the sum of aggregate consumption, (wasteful) spending by the government, investment (which includes adjustment costs), and intermediary expenditure incurred when liquidating bankrupt firms. We define GDP to be the sum of consumption, investment, and government spending, or equivalently, output minus deadweight losses.

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights ℓ:

$$
\mathcal{W}_t(\cdot; \Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^I V_t^I,
$$

where the $V^j(\cdot)$ functions are the value functions defined in the appendix.

3 Calibration

The model is calibrated at annual frequency. The parameters of the model and their targets are summarized in Table 1.

Aggregate Productivity We assume that aggregate productivity grows with a stochastic rate g_t that follows an AR(1) process:

$$
Z_{t+1} = \exp(g_{t+1}) Z_t \tag{19}
$$

$$
g_{t+1} = (1 - \rho_g)\bar{g} + \rho_g g_t + \epsilon_{t+1} \qquad \epsilon_{t+1} \sim iid\, \mathcal{N}(0, \sigma_g) \tag{20}
$$

Table 1: Calibration

This table reports parameter values.

Par	Description	Value	Target
		Exogenous Shocks	
$\bar{g}$	mean TFP growth	2.0%	Mean rpc GDP gr 53-14 of 2.00%
σ_g	vol. TFP growth	1.50%	Vol hp-detr GDP 53-14 of 1.95%
ρ_g	persistence TFP growth	0.75	AC(1) hp-detr GDP 53-14 of 0.34
σ_ω	vol. idio. prod. shock	{0.1,0.17}	Corporate default rates
p^ω_{LL}, p^ω_{HH}	transition prob	0.2,0.99	Frequency and duration of credit crises
		Production, Population, Labor Income Shares	
ℓ^i	pop. shares $i \in \{S, B, I\}$	{69,28.3,2.7}%	Population shares SCF 95-13, QCEW 01-15
γ^i	inc. shares $i \in \{S, B, I\}$	{60,37.4,2.6}%	Labor inc. shares SCF 95-13, QCEW 01-15
α	labor share in prod. fct.	0.71	Labor share of output 0.66
ψ	marginal adjustment cost	12	Vol. investment-to-GDP ratio 53-14 of 1.23%
δ_K	capital depr.	7.5%	Capital depreciation BEA 53-14
		Corporate loans	
δ	average life loan pool	0.937	Duration Fcn. (Appendix B)
θ	principal fraction	0.582	Duration Fcn. (Appendix B)
Φ	maximum LTV ratio	0.80	Technical assumption
ζ	DWL of bankruptcy	0.5	Corporate loan and bond severities 81-15
		Preferences	
$\beta^B = \beta^I$	time discount factor B, I	0.95	FoF nonfinancial sectors leverage 85-14
$\sigma^B = \sigma^I$	risk aversion B, I	1	Standard value
$\nu^B = \nu^I$	IES B, I	1	Standard value
β^S	time discount factor S	0.985	Mean risk-free rate 85-14
σ^S	risk aversion S	20	Financial leverage 85-14 of 95.6%
ν^S	IES saver	4	Vol. risk-free rate 85-14
η	effort cost level	2	normalization
ϕ	effort ineffectiveness	0.9	Vol hp-detr cons 53-14 of 1.89%
		Government Policy	
τ	personal income tax rate	24.86%	BEA govt pers tax rev to GDP 53-14 of 17.30%
τ_Π	corporate income tax rate	20.00%	BEA govt corp. tax rev to GDP 53-14 of 3.41%
G^o	exogenous govt spending	17.58%	BEA govt. discr. spending to GDP 53-14
G^T	govt transfers to agents	3.14%	BEA govt. net transfers to GDP 53-14
κ	deposit insurance fee	0	Deposit insurance fee 97-06
ξ	margin	0.95	Basel II reg. capital charge (C&I loans)
σ_ρ	bank bankruptcy	10%	Technical assumption

Given the persistence of TFP growth, g_t becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst (1995). The procedure matches the mean of GDP growth, and the volatility and persistence of detrended GDP growth, which are endogenous, by choosing both the productivity grid points and the transition probabilities between them. Consistent with our model, our measurement of GDP excludes net exports, housing investment, changes in inventories, and government investment. We define the GDP deflator correspondingly. The observed real per capita GDP growth between 1953 and 2014 has a mean of 2.00%, which the model matches (2.01%). The volatility of detrended GDP is 2.13% and its persistence is 0.68. The model generates a volatility of 2.41% and a persistence of 0.53.

Idiosyncratic Productivity Shocks and Credit Crises We keep the mean of idiosyncratic productivity μ_ω constant at 1. We let the cross-sectional standard deviation of idiosyncratic productivity shocks $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ are a source of aggregate risk because they affect the optimal effort choice of borrowers. At higher levels of $\sigma_{t,\omega}$ it is more costly to achieve the same level of aggregate effort m_t (in utility terms). Further, fluctuations in $\sigma_{t,\omega}$ govern aggregate corporate credit risk since high levels of $\sigma_{t,\omega}$ cause a larger left tail of low-productivity firms that default. We refer to states with the high value for $\sigma_{t,\omega}$ as high uncertainty periods or *credit crises*. The latter label anticipates that high uncertainty periods endogenously generate higher corporate default rates. We set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.10, 0.17)$ and the deadweight losses of default to $\zeta = 0.5$ to match losses given default. Together, these three parameters are important drivers of the default rate and the severity rate (loss given default rate) in normal times and in credit crises. Our baseline model generates an average default rate of 2.54%, an average severity (loss given default) of 38.92%, and thus an average loss rate of 1.16%. Average default, loss, and severity rates are all close to the data ensuring that our model generates the right quantity of corporate default risk.[11]

[11] We look at two sources of data: corporate loans and corporate bonds. From the Flow of Funds, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1% and the average loss rate is 0.7%. Default and loss rates are much higher in the recessions of 1991, 2001, and 2007-09. In the 1991 recession, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%. The second source of data is Standard & Poors' default rates on publicly-rated

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^{ω}_{LL} and p^{ω}_{HH}, to match the frequency and length of credit crises. Thus, the model implies that not all recessions are credit crises, but all credit crises are recessions. Based on the historical frequency of financial crises in Reinhart and Rogoff (2009), we target a 10% probability of a credit crisis. Conditional on a crisis, we set the expected length to 2 years, based again on evidence in Reinhart and Rogoff.

Production We set the marginal adjustment cost parameter $\psi = 12$ in order to match the observed volatility of the ratio of investment to GDP of 1.23%. The model generates a value of 1.14%, which is close. We set the parameter α in the Cobb-Douglas production function equal to 0.71, which yields an overall labor income share of 66%, the standard value. We choose δ_K to match an annual depreciation of capital of 7.5%. This is the observed depreciation rate in the 1953-2014 BEA fixed asset data, calculated as the average ratio of depreciation of total private nonresidential fixed assets to the net stock of total private nonresidential fixed assets, both measured at current cost.

Population and Labor Income Shares To pin down the population shares of our three different types of households we turn to the Survey of Consumer Finance (SCF).[12] We define savers as those households who hold a low share of their wealth in the form of risky assets. In particular, we compute for each household in the survey the share of assets, net of all real estate, held in stocks or private business equity, considering both direct and indirect holdings of stock. Using this definition of the risky share, we then calculate the fraction of households whose risky share is less than one percent. This amounts to 69% of SCF households. The remaining 31% of households have a large risky asset share. We split them into 28.3% borrowers-entrepreneurs

corporate bonds for 1981-2014. The average default rate is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The average severity rate on S&P and Moody's rated defaults between 1985 and 2004 is 44%. Again, default and loss rates are higher during recessions. During the 2001 recession, for example, the default rate on high-yield bonds was 9.9% and the loss rate 5.4%.

[12] We use all survey waves from 1995 until 2013 and average across them.

and 2.7% financial intermediaries based on the share of employees that work in the financial sector, defined as "Securities, Investments" and "Credit Intermediation" from the Quarterly Census of Employment and Wages, averaged over the longest available sample 2001-2015. The population shares are used for the welfare calculations.

From the same QCEW data, we obtain the wage share for the intermediaries of 2.6%. The labor income share of savers in the SCF is 60%. The income share of the borrower-entrepreneurs must then be the remaining 37.4%. The income shares determine the Cobb-Douglas parameters γ_I, γ_B, and γ_S. By virtue of the calibration, the model matches basic aspects of the observed income distribution. It also matches the size of the U.S. intermediary sector.[13]

Corporate Loans In our model, a corporate loan is a geometric bond. The issuer of one bond (firm) at time t promises to pay the holder (intermediary) 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Given that the present value of all payments $(1/(1-\delta))$ can be thought of as the sum of a principal (share θ) and an interest component (share $1-\theta$), we define the book value of the debt as $F = \theta/(1-\delta)$. This book value of debt is used in the firm's collateral constraint. We set $\delta = 0.937$ and $\theta = 0.582$ to match the observed duration of corporate bonds. Appendix B.1 contains the details. The model's corporate loans have a duration of 7 years on average.

Borrowers can obtain a loan with principal value up to a fraction Φ of the market value of their assets. We set the maximum LTV ratio parameter $\Phi = 0.8$. Corporate leverage is limited endogenously in the model because borrowers are optimally trading off the benefit of greater borrowing today against the cost of more defaults tomorrow. We set Φ to a value that is just large enough such that the LTV constraint of borrowers never binds during expansions and non-financial recessions. In the simulation of our benchmark model the constraint binds in 15% of all crisis periods (recession with high uncertainty).[14]

[13] Intermediaries' labor income is 2.6% of total labor income and 1.72% of GDP pre-tax and 1.27% post-tax. After-tax profits are an additional 1.28% of GDP. Total post-tax intermediary income is 2.55% of GDP in the model. The market value of intermediated assets is 67% of GDP. Thus, intermediary income is 3.8% of intermediated assets. Intermediary profits are 1.9% of intermediated assets. Philippon (2015) reports that the cost of financial intermediation has historically been about 2% of intermediated assets.

[14] Even though the maximum LTV constraint binds infrequently, we found that having a hard upper bound on corporate leverage was useful to obtain a stable numerical solution to the model.

Preference parameters Preference parameters are harder to pin down directly by data since they affect many equilibrium quantities and prices simultaneously. In order to highlight the separate roles of intermediaries' and firms' balance sheets, we purposely set the time discount factor and the risk aversion coefficient of borrowers and intermediaries equal. For simplicity, we assume that borrowers and intermediaries have log utility ($\sigma_B = \sigma_I = 1$). The subjective time discount factors $\beta_B = \beta_I = 0.95$ are set to target borrower leverage. The model generates average borrower leverage of 52.5%. In the Flow of Funds data, the average ratio of loans and debt securities of the nonfinancial corporate and nonfinancial noncorporate businesses to their non-financial assets is 37%, a lower value. [15]

Turning to saver preference parameters, given $\beta_I = 0.95$ and $\sigma_I = 1$, we set saver risk aversion to $\sigma_S = 20$ to match intermediary leverage. The average ratio of total debt to total assets for 1985-2014 is 90.7%.[16] The model generates average intermediary (book) leverage of 90.6%.

The time discount factor and risk aversion of the saver disproportionately affect the mean short-term interest rate and its volatility. We set $\beta^S = 0.985$ to generate a low mean real rate of interest of 1.78%. The intertemporal elasticity of the saver ν_S controls the volatility of the real interest rate which is 2.23% in the model. In the data, the mean real interest rate is 1.2% with a volatility of 2.0% over the period 1985-2014.[17]

Finally, we have to set the two preference parameters η and ϕ that govern the effort choice of borrowers. While η pins down the average level of m_t, ϕ can be understood as an elasticity of effort to fluctuations in idiosyncratic volatility. Therefore, η is a normalization and we set it to a numerically convenient value of 2. To pin down ϕ, we target the volatility of detrended consumption of 1.87% in the data. The model produces a higher consumption volatility of

[15]For the Flow of Funds leverage data, we use the post-1987 sample. Only in this sample is nonfinancial leverage stationary. Our model certainly misses some reasons for firms to hold more cash (negative debt) such as international tax reasons.

[16]Krishnamurthy and Vissing-Jorgensen (2015) identify a group of financial institutions as net suppliers of safe, liquid assets. This group contains U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations, Money market mutual funds, GSEs, Agency- and GSE-backed mortgage pools, Issuers of ABS, and REITs. The group of excluded financial institutions are Insurance Companies, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds.

[17]To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of Professional Forecasters.

2.84%.

Government parameters Our goal is to capture average government spending and tax revenues as well as their cyclical properties. The model has two sources of government spending and two sources of tax revenue.

Discretionary and transfer spending as a fraction of GDP are modeled as follows: $G^i_t/Y_t = G^i \exp\{b_i(g_t - \bar{g})\}, i = o, T$. The scalars G^o and G^T are set to match the observed average discretionary spending to GDP of 17.58% in the 1953-2014 NIPA data, and transfer spending to GDP of 3.18%, respectively.[18] We set $b_o = -0.5$ and $b_T = -5.5$ in order to match the slope in a regression of log spending to GDP on GDP growth and a constant. We closely match these slopes: -7.86 and -0.71 in model versus -7.26 and -0.74 in the 1953-2014 data.

Similarly, we model the labor income tax rate as $\tau_t = \tau \exp\{b_\tau(g_t - \bar{g})\}$. We set the tax rate $\tau = 26.21\%$ in order to match observed average income tax revenue to GDP of 17.30%.[19] We set the sensitivity of the tax rate to aggregate productivity growth $b_\tau = 20$ to match the observed sensitivity of log income tax revenue to GDP to GDP growth.

Fourth, we set the corporate tax rate that both financial and non-financial corporations pay to a constant $\tau_\Pi = 15.64\%$. This allows us to match observed corporate tax revenues of 3.41% of GDP. The model generates 3.38%. The tax shield of debt that firms and banks enjoy in the model reduces the tax they pay.

The final source of government spending is interest service on the debt, which is endogenous since both quantity and price of government debt are determined in equilibrium. In the data, net interest payments on government debt average to 2.98% of GDP.[20] This number is close to

[18]We divide by $\exp\left\{b_i/2\sigma_g^2/(1-\rho_g^2)(b_i-1)\right\}$, a Jensen correction, ensure that average spending means match the targets.

[19]We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

[20]Net interest expenses are interest payments to persons and businesses (line 28) minus income receipts on asses (line 10).

the observed average budget deficit of 3.04% of GDP. We do not aim to match this number since the government cannot run a 3% deficit in perpetuity lest the debt explode. In our calibration, the personal and corporate tax revenue is very close to the discretionary and transfer spending; the primary deficit averages close to 0% of GDP. Government debt to GDP averages 83% of GDP in a long simulation of the benchmark model. While it fluctuates meaningfully over prolonged periods of time (standard deviation of 52%), the government debt to GDP ratio remains stationary.[21]

We can interpret the risk-taker borrowing constraint parameters, ξ, as a regulatory capital constraint set by the government. Under Basel II and III, corporate loans and bonds have a risk weight that depends on their credit quality. For a 40% loss given default, the risk weight on C&I loans with 2.5 year maturity ranges from 13% for AAA, 54% for BBB-, 125% for B+, to 325% for CCC. A blended regulatory capital requirement of 5% (8% times a blended risk weight of 62.5%) seems appropriate. This implies that $\xi = 0.95$.

We set the deposit insurance fee parameter $\kappa = 0$ to reflect the fact that banks were not required to pay any deposit insurance fees between 1997 and 2006.[22]

Utility cost of risk-taker bankruptcy The model features a random utility penalty that intermediaries suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty at least some of the time. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.10$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of intermediaries) depends on the frequency of

[21]In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates τ_t when debt-to-GDP falls below $\underline{b^G} = 0.1$ and by gradually increasing personal tax rates when debt-to-GDP exceed $\overline{b^G} = 1.2$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -0.1. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 160%. Our simulations never reach the -10% and +160% debt/GDP states. The simulation spends 12% of the time in the profligacy and 32% of the time in the austerity region. Profligacy and austerity tax policies do not affect the amount of resources that are available for private consumption in the economy.

[22]FDIC premia were raised after the crisis. Well capitalized banks currently pay 2.5 cents per $100 insured.

credit crises, and the endogenous choices (asset and liability choice) of the intermediaries.

4 Results

We present results from a long simulation of the model (10,000 years). For all variables of interest, we report averages and standard deviations over time, as well as averages conditional on being in a good state (positive TFP growth and low uncertainty, i.e. $\sigma_{\omega,L}$), non-financial recession (negative TFP growth, low uncertainty), and financial recession (negative TFP growth and high uncertainty $\sigma_{\omega,H}$). We start by presenting macroeconomic moments, before turning to balance sheet variables and prices.

4.1 Macro Quantities

The model matches the 2.0% growth rate of GDP. All other macro-economic quantities in the model grow at the same 2.0% rate.

The first four rows of Table 2 report hp-detrended log macro quantities in data and model. GDP has similar volatility in data (2.13%) and model (2.41%). Investment is more volatile than GDP, and equally volatile in the data (6.31%) as in the model (6.15%). The model also does a good job matching the volatility of the investment rate (0.89% vs. 0.88%) and the investment share of GDP (1.23% vs. 1.14%). Hp-detrended consumption is less volatile in the data (1.87%) than in the model (2.84%). The same is true of the consumption share of GDP. Government spending has a volatility of 3.73% in the data and 1.75% in the model. Investment, consumption, and government spending are similarly pro-cyclical in model and data.

The model generates a substantial amount of persistence in detrended GDP of 0.53. To understand this persistence, it is useful to decompose log GDP into its components, using equation (2):

$$\begin{aligned}\log(GDP) &= \log(Y_t) + \log\left(1 - \frac{DWL_t}{Y_t}\right) \\ &= \alpha\log(Z_t) + (1-\alpha)\log(K_t) + \log(m_t) + \log\left(\frac{GDP_t}{Y_t}\right)\end{aligned}$$

Table 2: Unconditional Macroeconomic Quantity Moments

	mean	stdev	output corr.	AC
		Data		
GDP^{hp}		2.13%	1.00	0.68
INV^{hp}		6.31%	0.77	0.58
$CONS^{hp}$		1.87%	0.91	0.65
GOV^{hp}		3.73%	0.46	0.78
INV/K	10.51%	0.89%	0.44	0.82
INV/GDP	13.29%	1.23%	0.19	0.87
		Model		
GDP^{hp}		2.41%	1.00	0.53
INV^{hp}		6.15%	0.59	0.06
$CONS^{hp}$		2.84%	0.90	0.61
GOV^{hp}		1.75%	0.70	0.27
INV/K	9.59%	0.88%	0.37	0.65
INV/GDP	21.16%	1.14%	0.14	0.33

where the last line omitted the constant labor input term $\alpha \log(\bar{L})$. The persistence of GDP in the model is driven by the persistence of labor-augmenting productivity Z, whose autocorrelation is 0.82, as well as by the persistence of detrended capital, with AC of 0.69, and detrended managerial effort, with AC of 0.48. The ratio of GDP to output has low persistence of 0.17. Because DWL are volatile, they reduce the persistence of GDP. Indeed, output has a persistence of 0.92, while the persistence of GDP is 0.51. Capital and managerial effort depend on all the state variables of the model, and contribute importantly to the persistence of GDP. The volatility of the four hp-detrended components of GDP is 1.52%, 1.31%, 0.92%, and 2.01%, respectively.

Table 2 shows that the model generates the right amount of persistence in aggregate consumption, but too little persistence in aggregate investment. The model does a better job matching the persistence of investment relative to the capital stock and relative to GDP. The model finds a middle ground between overstating the average level of investment to GDP and understating the average investment-to-capital ratio. We conclude that the model delivers about the right quantity of macro-economic risk, on average across states of the world.

How deep are the recessions that the model generates? Non-financial recessions, defined as periods of low TFP growth and low uncertainty, have GDP that is 0.83% above its long-term mean, on average. In contrast, financial recessions, defined as low TFP growth accompanied by high uncertainty, have GDP that is 3.53% below trend on average. Similarly, GDP growth between year $t-1$ and year t is 2.13% if year t is a non-financial recession and -1.47% if year t is a financial recession. Investment falls 6.78% below trend in a typical financial recession. Aggregate consumption is 3.67% below trend.

Deep recessions in the data are often associated with credit crises (Reinhart and Rogoff 2009). To understand the dynamics of the model and how they compare to the experience of the years 2006–2015 where we had a credit boom followed by a financial crisis, we compute the following impulse-responses. We start off the model in year 0 in a high-growth state, the highest of five points on the TFP growth grid, and in the low uncertainty state ($\sigma_{\omega,L}$). In period 1, the model undergoes a change to the lowest TFP growth grid point. In one case (red line), the recession is accompanied by a switch to the high uncertainty state ($\sigma_{\omega,H}$). This is what we called a financial recession or credit crisis. In the second case, the economy remains in the low uncertainty state; a non-financial recession (blue line). From period 2 onwards, the two exogenous state variables follow their stochastic laws of motion in each case. For comparison, we also show a series that does not undergo any shock in period 1 but where the exogenous states stochastically mean revert from the high productivity growth state in period 0 (black line). For each of the three scenarios, we simulate 10,000 sample paths of 25 years and average across them. For ease of readability, the variables are normalized to 100 in year 0.

Figure 2 plots the macro-economic quantities, all detrended at their long-term growth rate of 2%. The top left panel is for the productivity level Z. By construction, it falls by the same amount in financial and non-financial recessions. The initial drop in productivity is about 3%. Because of the persistence in productivity growth, aggregate productivity Z continues to fall to a level about 10% below the peak level (after detrending). The black line shows how productivity would have evolved absent a shock in period 1. Again, because of persistence in growth rates, productivity stabilizes at a level about 5% above the time-0 value (after detrending). The other three panels show that detrended GDP, consumption, and investment. All fall precipitously in the first period of the recession from their peak values. The drop is much larger than that in productivity alone. The figure shows a much deeper recession when the economy is additionally hit by an uncertainty shock (red line) than if it is not (blue line). Detrended

GDP falls by 12% in a financial recession in the initial period in a financial recession. GDP barely changes in a non-financial recession because aggregate consumption offsets the decline in investment. Importantly, GDP remains lower for longer in a financial recession. It takes more than 10 years until the level of GDP catches up to the level typical of the recovery after a regular recession. Aggregate consumption shows a larger and more persistent gap between financial and non-financial recessions. It takes consumption fifteen years to return to the level associated with non-financial recessions following a financial recession. Finally, aggregate investment falls sharply in the first period of the financial crisis. In year one, investment is 40% below its peak level. Investment rebounds sharply to 10% below the peak level in the year after the crisis as the market value of installed capital, Tobin's q, recovers (see below).

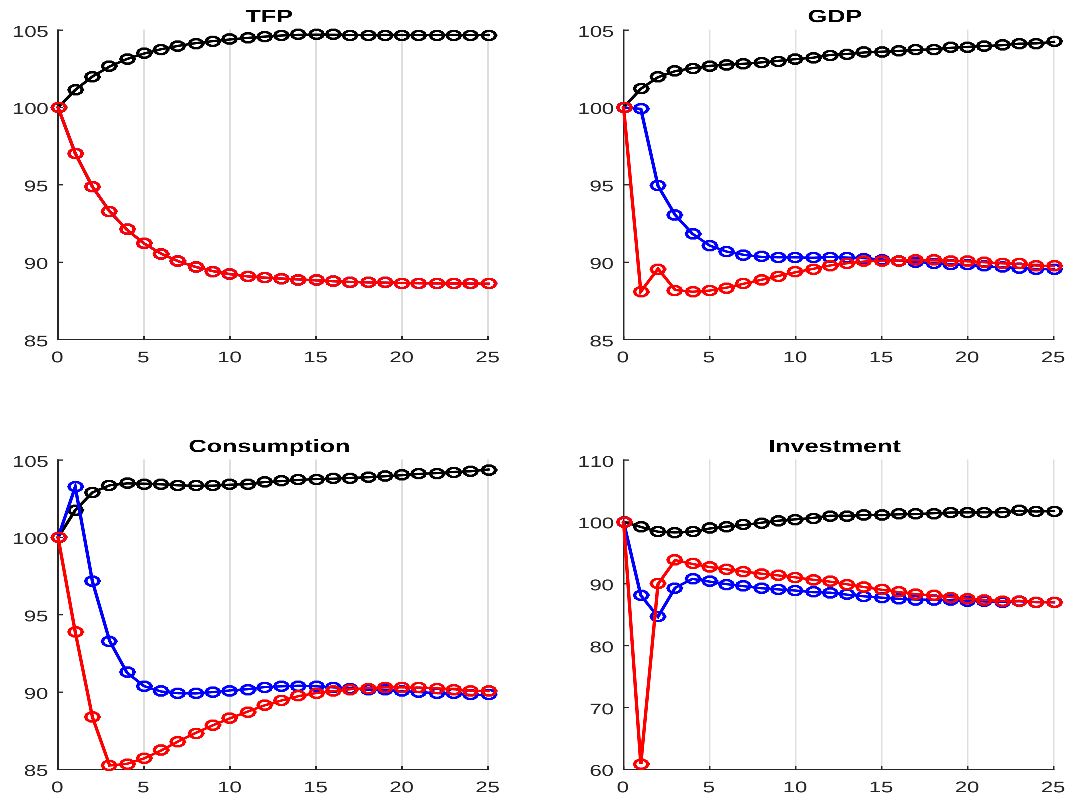
4.2 Balance Sheet Variables

Next, we turn to the key balance sheet variables in Table 3. The table reports the unconditional mean and volatility across a long simulation, as well as conditional averages in expansions, non-financial recessions, and financial recessions.

Firms The first two rows of the borrower panel show the market value of assets ($p_t K_t^B$) and the market value of liabilities of the non-financial corporate sector ($q_t^m A_t^B$), both scaled by GDP. Their difference is the market value of firm equity scaled by GDP. Their ratio is the market leverage ratio (row 4). Entrepreneurs own slightly less than half of their firms in the form of corporate equity. Total credit to non-financial firms amounts to 118% of GDP and non-financial corporate equity is 108% of GDP, on average. The volatility of the market value of firm assets relative to GDP is 19%, in line with the corporate finance literature.

The corporate leverage ratio is pro-cyclical in the model, both in market and in book value terms. In financial recessions, firm assets and liabilities drop substantially. In market value terms, they fall by 30% and 25% of GDP compared to expansions, respectively. The shrinkage in firm liabilities is 21% of GDP in book value terms. While the market price of corporate liabilities falls only modestly, consistent with the increase in the corporate loan/bond rate (reported in row 17), the market price of corporate assets falls substantially (row 15). Tobin's q falls by 30%, from 1.087 in expansions to 0.836 in financial recessions. This drop in prices accounts for

Figure 2: Financial vs. Non-financial Recessions: Macro Quantities



The graphs show the average path of the economy through a recession episode which starts at time 1. In period 0, the economy is in a high growth state, the second-highest point on the g-grid. The recession is either accompanied by high uncertainty (high σ_ω), a financial recession plotted in red, or low uncertainty (low σ_ω), a non-financial recession) plotted in blue. From period 2 onwards, the economy evolves according to its regular probability laws. The black line plots the We obtain these via a Monte Carlo simulation of 10,000 paths of periods 2-10, and averaging across these paths. **Blue line:** non-financial recession **Red line:** financial recession.

the entire drop in the market value of assets. Firm equity, falls only modestly from expansions (110% of GDP) to financial recessions (104.6% of GDP). The assets of the non-financial sector shrink by much more (-19%) than their equity (-9%), allowing firm leverage to be lower in a financial recession than in an expansion. In contrast, non-financial recessions see much smaller declines in the size of the corporate sector relative to GDP than financial recessions. The price of capital also falls much less. Non-financial recession see increases in firm equity relative to GDP, and the lowest average leverage of the three macro-economic states.

Average corporate leverage is 52.5% with a standard deviation of 6.8%. How much leverage firms take depends on the degree of patience and risk aversion of the firm owners, the advantageous tax treatment of debt, and the loss of collateral in case of a bankruptcy. The model generates a reasonable, interior amount of debt. Borrowers' maximum leverage constraint never binds in expansions and rarely in non-financial recessions (0.02%). It binds in 15.2% of the financial recessions. In financial recessions, the market value of firm collateral falls precipitously and borrowing constraints tighten. This reduces firm's debt capacity and, if the constraint binds, it forces firms to cut its borrowing from the financial sector. Most of the time, firms choose to stay away form the leverage constraint because they are risk averse and take into account the costs of bankruptcy.

Borrowers default when their profits are negative. This is more likely when the cross-sectional distribution of idiosyncratic productivity shocks widens, as the mass of firms with productivity shocks below the threshold ω_t^* increases. The model generates average corporate default and loss rates of 2.54% (row 6) and 1.16% points (row 8), respectively, implying an average loss-given-default rate of 39% (row 7). All these numbers are in line with the data. Default and loss rates are five times higher in financial recessions (7.5% and 3.9%) than in non-financial recessions and expansions (about 1.6% and 0.66% in both). The model generates the right amount of corporate credit risk, on average, and generates the strong cyclicality in the quantity of risk observed in the data.

How do firms' balance sheets respond in a financial crisis? The top row of Figure 3 plots the book value of corporate assets (left), the book value of corporate debt (middle), and the market value of corporate equity (right). The bottom left graph plots the market value of corporate debt.[23] The first two panels show that firms shrink their assets and liabilities much more in a financial recession than a non-financial recession. Over periods one and two combined, firms' debt falls by roughly the same percentage in market value (bottom left panel) and in book value terms (top middle panel). The market value of assets falls more sharply than the book value because of a sharp decline in the price of capital assets. As a result, the net worth of the corporate sector falls precipitously in period 1 (top right panel). Most of the sharp recovery in net worth that occurs in period 2 is the result of a rebound in the price of capital and a more modest decline in the market price of corporate liabilities. After period 2, as the economy normalizes and firms resume investment, corporate credit starts to recover gradually. It takes

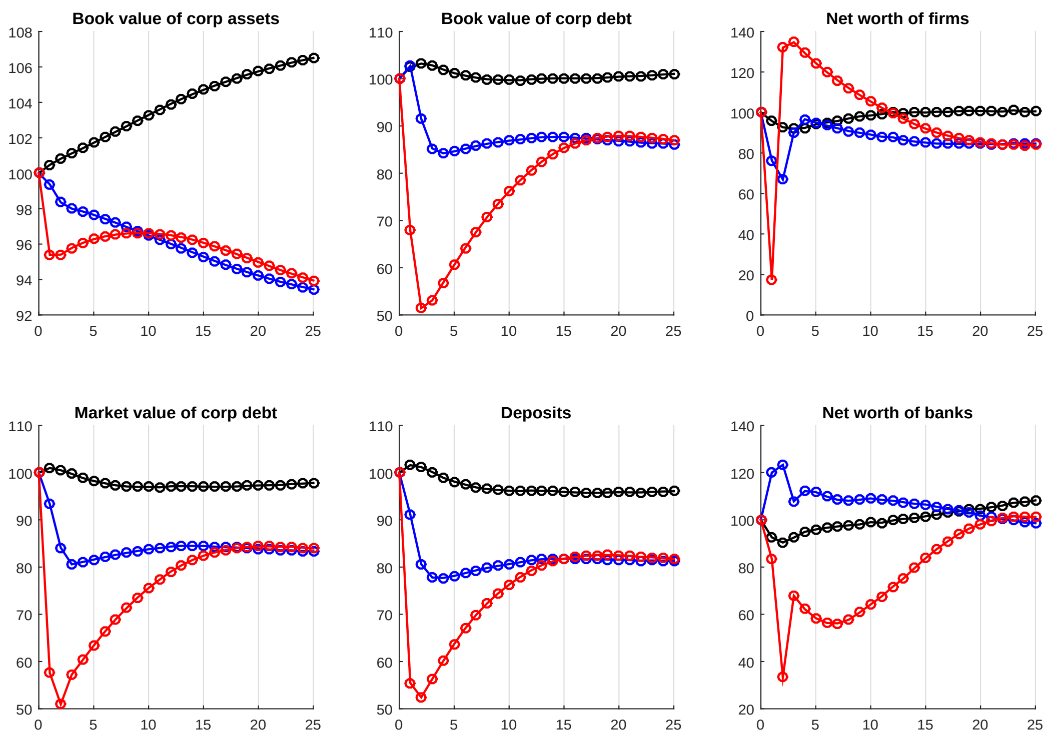
[23]All balance sheet variables are measured at the end of the period so that the shock at time 1 affects them in period 1 of the transition graph.

Table 3: Balance Sheet Variables and Prices

	Unconditional		Expansions	Non-fin Rec.	Fin Rec.
	mean	stdev	mean	mean	mean
			Borrower		
1. Mkt value of capital/GDP	2.253	0.194	2.358	2.249	2.055
2. Mkt Value of corp debt/GDP	1.174	0.123	1.257	1.173	1.009
3. Book value of corp debt/GDP	1.179	0.131	1.265	1.163	1.057
4. Market corp leverage	52.49%	6.80%	53.36%	52.31%	51.35%
5. Book corp leverage	52.67%	6.61%	53.72%	51.90%	53.17%
6. Default rate	2.54%	2.52%	1.69%	1.61%	7.46%
7. Loss-given-default rate	38.92%	12.95%	37.66%	39.06%	40.98%
8. Loss Rate	1.16%	1.97%	0.66%	0.66%	3.87%
			Intermediary		
9. Mkt fin leverage	89.06%	4.18%	90.14%	87.41%	92.53%
10. Book fin leverage	90.62%	4.04%	92.45%	89.10%	92.09%
11. Fraction leverage constr binds	12.58%	33.16%	14.49%	1.85%	45.51%
12. Bankruptcies	1.84%	13.44%	0.32%	2.32%	3.30%
			Saver		
13. Deposits/GDP	1.064	0.126	1.162	1.045	0.930
14. Government Debt/GDP	0.830	0.522	0.593	0.885	1.127
			Prices		
15. Tobin's q	1.001	0.117	1.085	0.999	0.835
16. Risk-free rate	1.78%	2.23%	2.58%	1.93%	-0.36%
17. Corporate bond rate	4.47%	0.40%	4.49%	4.34%	4.85%
18. Credit spread	2.68%	2.20%	1.91%	2.41%	5.21%
19. Excess return on corp. bonds	1.54%	3.84%	1.24%	2.30%	-0.49%

15 years following a financial recession before corporate credit has caught up with the level of credit that follows a non-financial recession. In sum, the model generates a persistent credit slump.

Figure 3: Financial vs. Non-financial Recessions: Balance Sheet Variables



Blue line: non-financial recession **Red line:** financial recession.

Intermediaries Intermediary leverage is 89.0% in market value and 90.6% in book value terms on average in the model. It matches the 90.7% financial leverage target. Intermediaries choose to be so highly levered for a number of reasons. Like the corporate firms, they are impatient and enjoy a tax shield. As the only agent with access to deposits, they alone can earn a large spread (2.7%, row 18) between the short-term deposit rate (1.8%, row 16) and the rate on corporate loans (4.5%, row 17). They bear the interest rate risk associated with the maturity transformation they perform, as well as the credit risk on the loans. Given the low (but realistically calibrated) average loss rate, they choose to take up substantial leverage to reach their desired risk-return trade-off.

Intermediary leverage is lower in non-financial recessions and higher in financial recessions than in an average period. The behavior of market leverage for financial intermediaries over the cycle is opposite that of non-financial firms. The main reason for the rise in intermediary leverage in financial recessions is that the market value of banks' assets, corporate loans, falls substantially. This occurs because default risk and default risk premia rise, both of which increase corporate bond yields. Banks also suffer losses on their corporate loan portfolio (line 19). At the same time that the value of bank assets shrinks, their liabilities shrink. Bank liabilities fall from 116% of GDP in expansions to 93% of GDP in financial recessions. Since GDP itself falls, bank liabilities themselves fall by as much as 29%.

A lower value of bank assets tightens their regulatory capital constraint. The intermediary leverage constraint binds in 45.5% of the financial crises compared to 12.6% unconditionally. When binding, intermediaries must reduce liabilities to meet capital requirements in the wake of their credit losses. Given the low cost of deposit funding in a financial crises (-0.36%) and the high credit spreads they earn in those states of the world (5.21%), intermediaries would like to raise more deposits and increase corporate lending but their constraint prevents them from doing so.

In sharp contrast, intermediaries are unconstrained in all but 1.9% of the non-financial recessions. They earn their desired expected return per unit of risk in such periods. Given the available investment opportunities, they do not desire to expand their corporate lending to the point where there leverage constraint would bind. Intermediaries are constrained in 14.5% of the expansions when investment opportunities are good and they would like to expand their lending.

Intermediary net worth, or bank equity, is an important state variable in all intermediary-based models. Intermediary net worth is the difference between the market value of bank assets (row 2) and the book value of deposits (row 13). Intermediary equity to GDP is 11% unconditionally. It shrinks to 7.9% of GDP in financial recessions. The reduction in intermediary net worth itself is 24%, going from expansion to non-financial recession. Intermediary equity is 44% lower in financial recessions than unconditionally. The reduction in net worth in financial crises makes intermediaries effectively more risk averse, leading them to charge larger risk premia on new lending. We return to this risk premium effect below. Low net worth hampers the intermediaries' capacity to bear credit risk and do maturity transformation. Interestingly, intermediary equity is highest and leverage lowest in non-financial recessions.

In the equilibrium of our model, the intermediation sector is insolvent in 1.8% of the periods. Bank bankruptcies are concentrated in financial crisis, when they occur 3.3% of the time (row 12). In those periods, the government steps in, makes whole the depositors (short-term creditors of the financial sector), and takes over the assets of the banks at their market value. The banking sector restarts with zero wealth (and its labor endowment) the next period. Deposit insurance lowers the cost of funding and provides banks with a risk shifting motive vis-a-vis the government. However, as risk averse agents, bank owners are reluctant to hit low net worth states since they imply low consumption and high marginal utility. The balance of these two factors generates rare financial disasters.

The bottom panels of Figure 3 show the impulse-response functions for banks' assets and liabilities for the boom-bust exercise. The market value of banks' assets is in the bottom left (being equal to the market value of corporate debt), while the deposits in the middle panel are the banks' liabilities. The difference between the two is the banks' net worth, plotted in the bottom right panel. A financial crisis coincides with a sharp drop in both bank assets and liabilities, a 40% drop from the peak level. Bank net worth falls only modestly in the first period. In period 2 bank assets decline further, while bank liabilities remain at their already depressed level, and financial intermediary net worth falls sharply. Cumulatively, banks loose nearly 2/3 of their net worth in two years. There is a small rebound in the market value of banks' assets in year 3, which is largely driven by an increase in the price of corporate loan assets. The recovery in loan values relaxes banks' borrowing constraint and allows them to increase deposits. The quantity of loans and deposits then gradually grows back to the level of the non-financial recession over the course of the next 18 years. Constrained intermediaries are unable to take advantage of the low deposit rate and high loan spreads which prevails at the onset of the crisis. It takes many years for intermediaries to rebuild their net worth to pre-recession levels, and to bear the levels of aggregate risk they bear in a boom when they are well capitalized. The slow recovery in intermediary wealth is a key factor in the slow macro-economic recovery.

Savers Risk averse savers only hold safe debt, provided both by the intermediaries and the government. On average, these two sources of safe assets account for 106.4% (row 13) and 83.0% of GDP (row 14). In our model, as in the data, the government's tax revenues are pro-cyclical and its expenses counter-cyclical. The counter-cyclical primary deficit to GDP makes that government debt is a larger fraction of GDP in recessions than in expansions. This is especially true in financial recessions, which are deeper and may involve a financial sector bailout. Government debt is 113% of GDP in financial recessions and 59% of GDP in expansions. The expansion in government debt to GDP more than offsets the reduction in deposits so that the total equilibrium supply of safe short-term assets rises in financial crises. The government's share in the supply of safe assets increases from 34% in expansions to 55% in financial crises. Despite the increase in the supply of safe assets in crises, the equilibrium real interest rate falls nearly 300 basis points. The reason is the strong increase in demand for safe assets from the saver, who is the marginal agent absorbing fluctuations in safe assets. In financial crises, savers' precautionary motives strengthen.

Prices Real interest rates on safe debt are 1.78% on average and have a volatility of 2.23% (row 16). Both are very reasonable numbers, especially in a production-based asset pricing model, and match historical averages. Financial recessions see large declines in collateral values (row 15), negative (excess) returns on bank assets (row 19), high corporate credit spreads (row 18), and very low real interest rates (row 16). All of these are important features of real-life financial crises. In contrast, non-financial recessions see only modest reductions in risk-free rates and modest increases in credit spreads.

One important quantitative success of the model is its ability to generate a high unconditional credit spread while matching the observed amount of default risk. The average 2.68% spread is close to the average 2.39% spread in the data.[24] The credit spread is also highly volatile (2.2%

[24]We define the credit spread as the difference between the yield on a blended portfolio of investment grade and high yield bonds and the yield on a one-year constant maturity Treasury yield. We use the longest available sample from Barclays U.S. corporate IG and HY bond indices from February 1987 to December 2015. To determine the portfolio weights on the high yield versus investment grade bonds, we use market values of the amounts outstanding, also from Barclays. The average weight on HY is 19.4%. The resulting credit spread has a mean of 3.36% and a volatility of 1.46%. We compare this with a difference measure of the credit spread which takes a 19.4%-80.6% weighted average of the Moody's Aaa and Baa yields and subtracts the one-year CMT rate. Over the same February 1987-December 2015 period, the mean credit spread is 3.05% with a volatility of 1.39%. The second measure of the credit spread has a correlation of 86% with the first one. The advantage

standard deviation) and strongly counter-cyclical (three times higher in financial recessions than expansions). The rise in the credit spread in financial recessions reflects not only the increase in the default risk but also an increase in the credit risk premium. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical SDF of the intermediary, who is the marginal agent in the corporate loan market. The sharp decline in intermediary wealth is responsible for the sharp increase in the SDF.

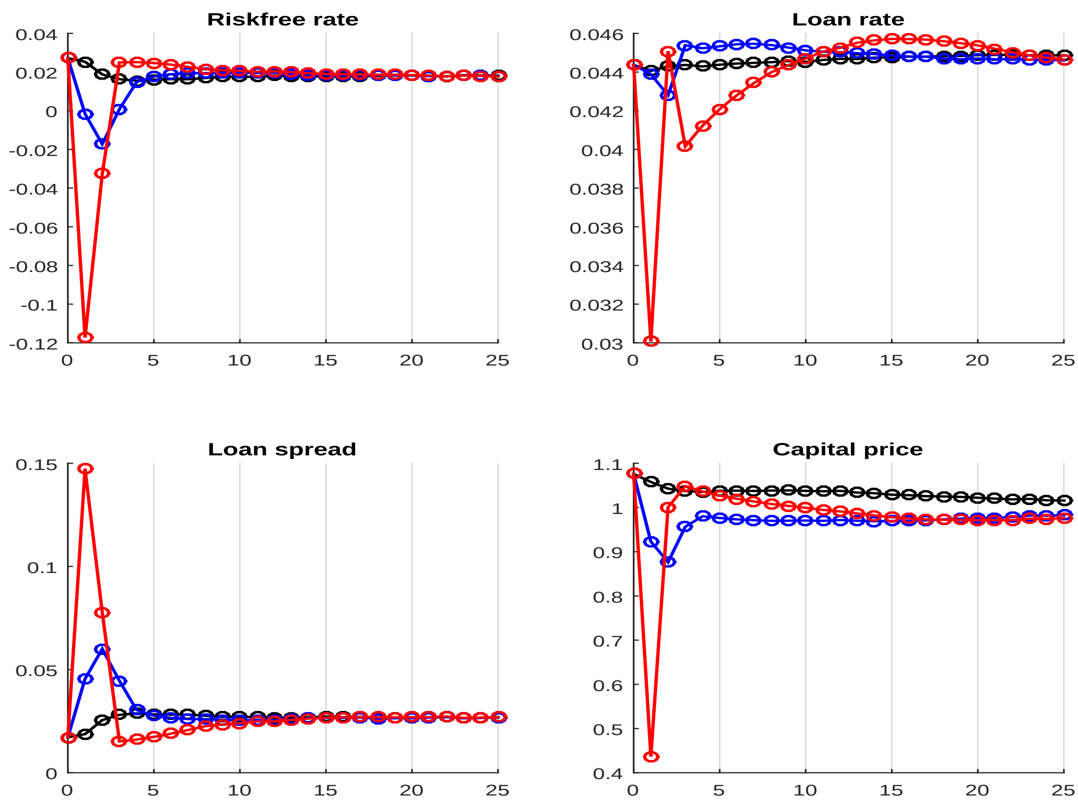
Figure 4 shows the impulse-responses for the interest rates, the credit spread, and the price of capital. In the first period of a financial recession following a boom, the real risk-free rate turns sharply negative and the credit spread blows out to 15%. Financial recessions are periods of high credit risk and credit risk premia, both of which enter in the credit spread. Strong precautionary savings motives depress the real rate. The price of capital falls by more than 50% in financial recessions, as the present discounted value of the marginal product of capital declines together with its value as a collateral asset. Non-financial recessions see a much smaller risk-free rate and credit spread effect, and a much smaller decline in the price of capital.

4.3 Consumption and Welfare

Table 4 reports the moments of consumption for each agent, as well as each agent's value function, and aggregate welfare. By virtue of being the largest groups of agents, borrowers and savers have the highest consumption shares (relative to GDP). More interesting is consumption growth in the second panel. It reveals that the intermediary has by far the most volatile consumption growth (48.7%), followed by the borrower (14.4%), and the saver (10.1%). We recall that borrowers and intermediaries have the identical preferences (risk aversion, patience, and IES), so that these differences in consumption volatility arise endogenously from the different roles they perform in the economy. Intermediaries end up absorbing a disproportionate fraction of the aggregate risk in the economy. In financial recessions, all three agents suffer drops in their consumption, but the drop is far larger for intermediaries (-20.0%) than for borrowers (-2.7%)

of this second measure is that we can compute it back to 1953. The mean spread over the 1953-2015 period is 2.08% with a volatility of 1.55%. While the second credit spread is downward biased, compared to the better first measure, considering a longer sample period would lead us to consider a lower mean credit spread target than 3.36%. For example, we could add the 31 basis point difference between our favorite Barclays measure and the Moody's measure over the post-1987 sample to the full-sample Moody's mean of 2.08% to get to a target mean credit spread for the full 1953-2015 sample of 2.39%.

Figure 4: Financial vs. Non-financial Recessions: Prices



Blue line: non-financial recession **Red line:** financial recession.

or savers (-1.3%). Financial recessions destroy wealth, but they also redistribute wealth from intermediaries to borrowers, who can default on their debt, and especially to savers.

The third panel reports moments related to aggregate welfare. Overall welfare, the population weighted average of the three agents' value functions and measured in consumption equivalent units, is highest in expansions and lowest in financial recessions. The welfare difference between expansions and financial recessions is 8.8% whereas it is only 5.1% between expansions and non-financial recessions. Part of the larger welfare loss comes from the increase in defaults in financial recessions, which cause a rise in the deadweight losses of bankruptcies. Intermediaries have 12% lower welfare in financial recessions than in expansions, while borrowers (-10%) and savers (-8.7%) suffer much less.

The last panel reports ratios of marginal utilities between pairs of agents. In a complete

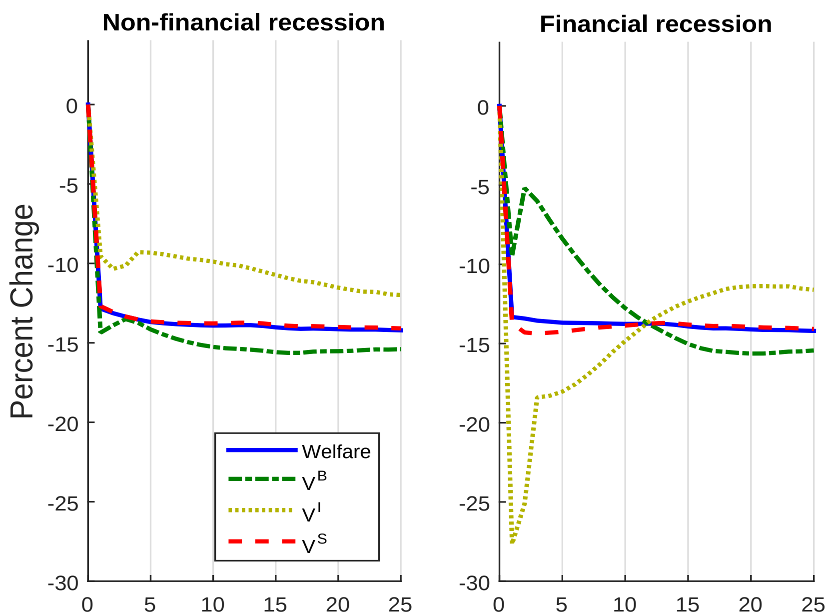
Table 4: Consumption and Welfare

	Unconditional		Expansions	Non-fin Rec.	Fin Rec.
	mean	stdev	mean	mean	mean
Consumption to Output					
Consumption, B	0.265	0.026	0.274	0.263	0.251
Consumption, I	0.024	0.006	0.023	0.027	0.016
Consumption, S	0.317	0.034	0.306	0.318	0.333
Consumption growth					
Consumption, B	2.01%	14.42%	4.39%	1.95%	-2.67%
Consumption, I	2.01%	48.68%	1.60%	8.62%	-19.85%
Consumption, S	2.01%	10.08%	2.76%	2.52%	-1.28%
Welfare					
Aggregate welfare	0.406	0.014	0.423	0.402	0.388
Value function, B	0.141	0.007	0.147	0.139	0.133
Value function, I	0.026	0.002	0.027	0.026	0.024
Value function, S	0.530	0.018	0.551	0.524	0.507
DWL/GDP	0.034	0.028	0.024	0.022	0.093
Marginal utility ratios					
log(MU B/MU I)	-0.762	0.364	-0.801	-0.628	-1.139
log(MU B/MU S)	0.055	0.196	-0.010	0.065	0.154
log(MU S/MU I)	-0.817	0.390	-0.791	-0.693	-1.293

markets model with agents whose preference parameters differ, these ratios would differ across pairs of agents but be constant over time. Our model is an incomplete markets model featuring imperfect risk sharing; the marginal utility ratios display high volatility. For example, the marginal utility of borrower and saver are nearly equalized in expansions, but the borrower has 15% higher marginal utility than the saver in financial recessions. Despite having identical preference parameters, the log difference of the borrower's and intermediary's marginal utility fluctuates between -63% and -114%.

Figure 5 shows the evolution of the value functions of the three types of agents and aggregate welfare for the boom-bust experiment. The main distinction between non-financial (left panel) and financial recessions (right panel) is that intermediaries suffer much greater welfare losses in a financial recession. The opposite is true for borrowers. Borrowers can recapitalize at the expense of the banks in financial recessions, shedding debt in bankruptcy and buying capital at distressed prices. This wealth distribution effect of financial recessions is clearly visible in the right panel. In contrast, borrowers suffer more than intermediaries in non-financial recessions. Savers lose somewhat more in financial recessions than in non-financial recessions due to the depressed interest rates.

Figure 5: Welfare: Non-financial vs. Financial Recessions



4.4 Credit Spread and Risk Premium

Figure 6 shows the histogram of the intermediary wealth share plotted against three different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield r_t^m on corporate bonds and the risk-free rate, where we compute the bond yield as $r_t^m = \log\left(\frac{1}{q_t^m} + \delta\right)$.[25]

Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.

[25] This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream $(1, \delta, \delta^2, \dots)$ occurring in the future.

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary's stochastic discount factor with the corporate bond's return, and an additional component that reflects the tightness of the intermediary's leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by the constrained intermediary, such that the market risk free rate is lower than the "shadow" risk free rate implied by intermediary consumption smoothing. To decompose the expected excess return into the risk premium and constraint tightness components, we plot both the complete EER on corporate bonds (the dashed red line) and the risk premium (the dotted red line). When the intermediary wealth share is relatively high, the leverage constraint is not binding and the EER is equal to the risk premium, which is approximately zero in this region of the state space. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during credit crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis when intermediary wealth reaches zero, the EER reaches 17 percent. The conventional risk premium (the covariance of intermediary SDF and bonds return) accounts for 4 percentage points of the EER. The remaining 13 percentage points reflect the difference between the market risk free rate determined by saver consumption growth and the much higher shadow risk free rate of the intermediary.

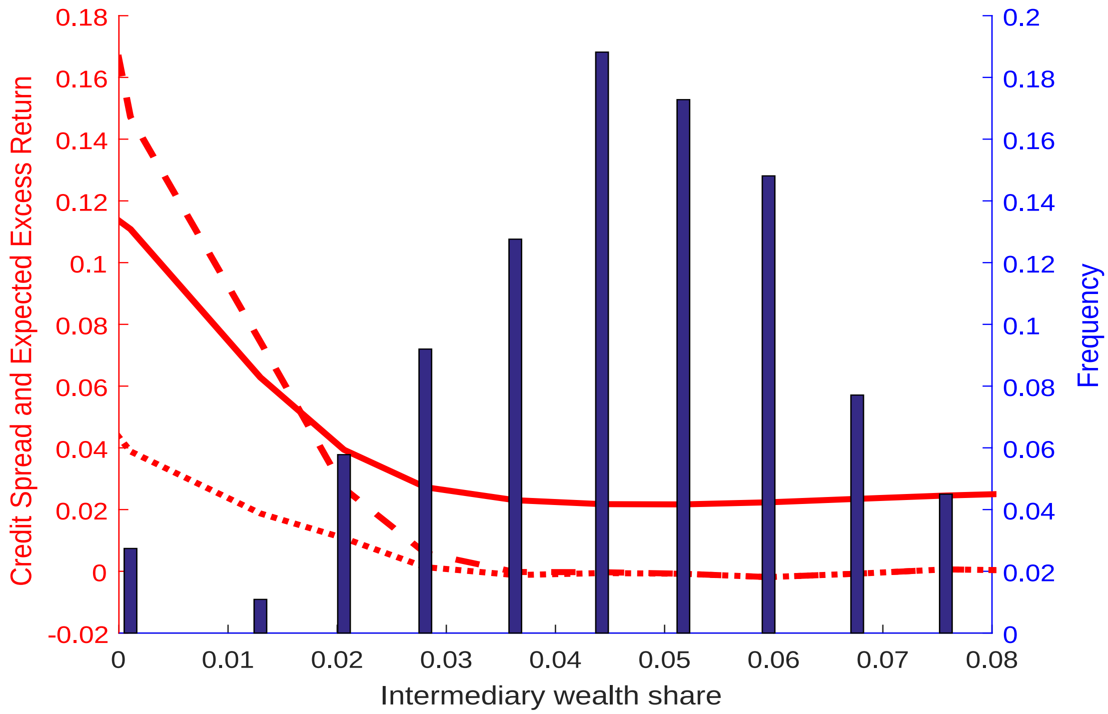
5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. Our main experiment is a variation in the intermediaries' leverage constraint. In the benchmark model, intermediaries can borrow 95 cents against every dollar in assets ($\xi = 0.95$). We explore tighter constraints ($\xi = .90$, $\xi = 0.80$), as well as looser constraints ($\xi = .975$). Table 5 shows that a tighter (looser) policy achieves its goal of reducing (increasing) financial sector leverage (rows 9 and 10). Bank liabilities shrink alongside bank assets when the leverage constraint becomes tighter. While small movements affect the degree of constraint bindingness little, when $\xi = .80$, the fraction of binding borrowing constraints doubles.

Table 5: Macroprudential Policy

	$\boldsymbol{\xi = 0.95}$	$\boldsymbol{\xi = 0.80}$	$\boldsymbol{\xi = 0.90}$	$\boldsymbol{\xi = 0.975}$
		Borrower		
1. Mkt value of capital/GDP	2.253	2.198	2.231	2.277
2. Mkt value of corp debt/GDP	1.174	0.866	1.124	1.201
3. Book val corp debt/GDP	1.179	0.943	1.160	1.178
4. Market corp leverage	0.525	0.392	0.506	0.532
5. Book corp leverage	0.527	0.430	0.523	0.521
6. Default rate	0.025	0.016	0.024	0.026
7. Loss-given-default rate	0.389	0.206	0.378	0.390
8. Loss Rate	0.012	0.004	0.010	0.012
		Intermediary		
9. Mkt fin leverage	0.891	0.725	0.829	0.926
10. Book fin leverage	0.906	0.733	0.844	0.943
11. Fraction lvg constr binds	0.126	0.380	0.144	0.145
12. Bankruptcies	0.018	0.021	0.005	0.032
		Saver		
13. Deposits/GDP	1.064	0.636	0.948	1.133
14. Government debt/GDP	0.830	0.711	0.797	0.867
		Prices		
15. Risk-free rate	0.018	0.012	0.018	0.019
16. Corp bond rate	0.045	0.055	0.048	0.042
17. Credit spread	0.027	0.043	0.030	0.023
18. Excess ret corp bonds	0.015	0.037	0.019	0.012

Figure 6: The Credit Spread and the Financial Intermediary Wealth Share



Because the intermediary is better capitalized with tighter macro-prudential policy, she is better able to absorb credit losses. Moreover, the loss rate falls as the intermediary constraint tightens. Notwithstanding the lower default risk, the intermediary earns a higher credit spread. This reflects the increased scarcity of intermediary capital in the economy with tight constraints. Because of the higher profit margin, the intermediary's wealth is higher as is her consumption share; see the middle panel of Table 6. Intermediary welfare increases with tighter constraints. Interestingly, it is the agent whose constraint is tightened, the intermediary, that benefits the most from the policy change. Macro-prudential policy redistributes wealth towards bankers.

Tighter bank capital requirements have a modest adverse effect on non-financial corporations. They face a higher corporate bond rates and reduce leverage. The effect is particularly pronounced for $\xi = .80$. With lower corporate leverage, loss rates are de minimis. The consumption share of and the welfare of borrower-entrepreneurs declines. Savers' welfare is affected adversely as well. There is less safe debt for them to hold in equilibrium, largely due to the tighter bank capital requirements. The equilibrium interest rate drops as a result, which reduces their return on investment and their consumption share.

A key effect of tighter macro-prudential policy is that the economy's output shrinks. The same is true for the capital stock. GDP shrinks by less than output because deadweight losses from bankruptcies fall with the incidence of bankruptcy as ξ is lowered. The reduction in output (and in GDP for low enough ξ) arises because firms are smaller and borrow less from the intermediary sector.

A second key effect of macro-prudential policies is that they reduce financial fragility and macro-economic volatility. The last panel of Table 6 indeed shows reductions in macro-economic volatility. Risk sharing improves as witnessed by the reduced volatility in the marginal utility ratios across pairs of agents.

The aggregate welfare effect of tighter bank capital requirements is modest. Tighter bank capital requirements reduce the risk in the economy but they shrink the total pie. On net, we find that tigher macro-prudential policy has small welfare losses, on the order of -0.23% consumption equivalent loss for a 5% point tightening of the bank leverage constraint. Higher leverage ($\xi = .975$) leads to a larger financial sector, more output, more bankruptcies, and higher welfare.

Table 6: Macroprudential Policy: Macro and Welfare

	$\boldsymbol{\xi = 0.95}$	$\boldsymbol{\xi = 0.80}$	$\boldsymbol{\xi = 0.90}$	$\boldsymbol{\xi = 0.975}$
		Welfare		
19. Aggregate welfare	0.406	-0.34%	-0.23%	+0.15%
20. Value function, B	0.141	+2.57%	-1.10%	+1.16%
21. Value function, I	0.026	+106.42%	+15.23%	-9.14%
22. Value function, S	0.530	-0.86%	-0.16%	+0.06%
23. DWL/GDP	0.034	-38.56%	-4.97%	+1.04%
		Size of the Economy		
24. Output	0.822	-2.07%	-0.14%	+0.23%
25. Capital stock	1.766	-3.39%	-0.97%	+1.25%
26. Consumption share, B	0.261	+0.62%	-1.26%	+0.72%
27. Consumption share, I	0.022	+37.83%	+14.83%	-8.17%
28. Consumption share, S	0.323	-2.06%	+0.67%	-0.80%
		Volatility		
30. GDP growth	2.60%	+41.26%	-5.11%	+5.37%
31. Consumption growth	2.82%	+123.99%	-2.23%	+43.63%
32. Investment growth	8.53%	+4.34%	-11.78%	+9.13%
33. Consumption growth, B	14.42%	+26.04%	-17.51%	+11.36%
34. Consumption growth, I	48.68%	+76.60%	-21.10%	+9.54%
35. Consumption growth, S	10.08%	+98.16%	-9.85%	+85.83%
36. log (MU B / MU S)	0.196	+27.12%	-20.74%	+22.99%
37. log (MU S / MU I)	0.391	+76.19%	-19.16%	+9.18%
38. log (MU B / MU I)	0.362	+30.96%	-25.25%	+13.02%

6 Conclusion

We provide the first calibrated macro-economic model which features intermediaries who extend long-term defaultable loans to firms producing output and raise deposits from risk averse savers, and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

Like in the standard accelerator model, shocks to the macro-economy affect entrepreneurial net worth. Since firm borrowing is constrained by net worth, macro-economic shocks are amplified by tighter borrowing constraints. Unlike the original models, ours features impatient but risk averse and infinitely-lived entrepreneurs. A second financial accelerator arises from explicitly modeling the financial intermediaries' balance sheet as separate from that of the entrepreneur-borrowers and saving households. Intermediaries are subject to regulatory capital constraints. Macro-economic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries' net worth and the indirect effect on borrowers to whom the intermediaries lend. However, when intermediaries are well enough capitalized to absorb the fundamental shock without constraining the firms, they can dampen the first accelerator mechanism.

We explore the dynamics of quantities and prices in this setting and compare them to U.S. data, with a focus on understanding differences between financial and non-financial recessions. Our main application studies macro-prudential policy and contrasts restrictions on firm leverage to those on bank leverage. While such policies reduce the credit risk and promote macro-economic stability and better risk sharing among the agents, they potentially shrink the size of the economy and may ultimately be welfare-reducing.

Extensions to this model could introduce New Keyenesian elements such as nominal rigidities, monopolistic competition, and monetary policy. Our setting is an interesting one to evaluate the effect of the zero lower bound (ZLB) on nominal short rates. A binding ZLB during a financial crisis would keep the real rate elevated. The ZLB economy would prevent the intermediaries from recapitalizing in a crisis through a negative real deposit rate. Negative real rates mitigate the severity of financial recessions in the current model. The upshot is that a binding ZLB may lead to a severe crisis exactly because it prevents a recapitalization of financial intermediaries.

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A Model Appendix

A.1 Contract between Family and Members

We restrict the contract between family and individual members to a schedule (C_0, C_1) that is increasing in observed productivity:

$$C_i = C_0\, C_1^{g(\omega_i, e_i)},$$

with $C_1 > 1$. We further assume that idiosyncratic shocks are distributed $N(\mu_\omega, \sigma_\omega^2)$. Each borrower household has preferences over consumption and effort:

$$u(\omega_i, e_i) = \log\left(C_0\, C_1^{g(\omega_i, e_i)}\right) - \eta e_i = c_0 + g(\omega_i, e_i)c_1 - \eta e_i,$$

where lower-case letter denote logs. The first-order condition for effort of borrower i is

$$e_i = \omega_i c_1^{1/\phi} \left(\frac{1-\phi}{\eta}\right)^{1/\phi}.$$

Consequently, idiosyncratic productivity as a function of the shock is

$$\hat{g}(\omega_i) = \omega_i c_1^{(1-\phi)/\phi} \left(\frac{1-\phi}{\eta}\right)^{(1-\phi)/\phi}.$$

Further, define average productivity as

$$m = \mathrm{E}_\omega[\hat{g}(\omega_i)] = \mu_\omega c_1^{(1-\phi)/\phi} \left(\frac{1-\phi}{\eta}\right)^{(1-\phi)/\phi},$$

and note that we can therefore express c_1 as function of average productivity and μ_ω

$$c_1 = \left(\frac{m}{\mu_\omega}\right)^{\frac{\phi}{1-\phi}} \frac{\eta}{1-\phi}.$$

To formulate the problem of the aggregate borrower family, first compute aggregate borrower consumption as

$$C = C_0 \int C_1^{\hat{g}(\omega)} dF(\omega) = C_0 \int \exp\left(c_1 \hat{g}(\omega)\right) dF(\omega).$$

To compute the integral, first write

$$
\begin{aligned}
c_1 \hat{g}(\omega) &= \omega c_1 c_1^{\frac{1-\phi}{\phi}} \left(\frac{1-\phi}{\eta}\right)^{\frac{1-\phi}{\phi}} \\
&= \omega c_1^{\frac{1}{\phi}} \left(\frac{1-\phi}{\eta}\right)^{\frac{1-\phi}{\phi}} \\
&= \omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} \frac{\eta}{1-\phi},
\end{aligned}
$$

where we used the expression for mean productivity to eliminate c_1. Since ω is normally distributed, the integrand is the expectation of a log-normal random variable, and one gets

$$
C = C_0 \exp\left[\mu_\omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} \frac{\eta}{1-\phi} + 0.5\left(\sigma_\omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} \frac{\eta}{1-\phi}\right)^2\right].
$$

Similarly, aggregate borrower utility is

$$
U = c_0 + c_1 \int \hat{g}(\omega) dF(\omega) - \eta \int e(\omega) dF(\omega) = c_0 + \int (c_1 \hat{g}(\omega) - \eta e(\omega)) dF(\omega).
$$

Computing the second term in the integrand

$$
e(\omega) = \omega c_1^{1/\phi} \left(\frac{1-\phi}{\eta}\right)^{1/\phi} = \omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}},
$$

gives

$$
U = c_0 + \mu_\omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} \left(\frac{\eta}{1-\phi} - \eta\right).
$$

To fully express utility as a function of aggregate consumption C and mean productivity m (foreshadowing that these will be the choice variables of the family), eliminate c_0 using the expression for aggregate consumption to get

$$
U(C, m) = \log(C) - \eta \mu_\omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} - 0.5\left(\sigma_\omega \left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} \frac{\eta}{1-\phi}\right)^2,
$$

This is log utility over aggregate family consumption with two additional terms that represent the penalty for effort from greater mean productivity m. The first term is the linear disutility from effort, which is scaled by the mean of the shocks. The second term is a Jensen correction term since borrowers are risk-averse, and it is relatively harder to incentivize borrowers with low realizations of ω_i to provide effort (the extra consumption that must be promised is increasing and convex in the decrease in ω). To embed this utility function in recursive Epstein-Zin preferences, we exponentiate to get

$$u(C,m) = C\exp\left[-\eta\mu_\omega\left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}} - 0.5\left(\sigma_\omega\left(\frac{m}{\mu_\omega}\right)^{\frac{1}{1-\phi}}\frac{\eta}{1-\phi}\right)^2\right],$$

which is the function given in equation (5).

Aggregate production is

$$\int_\Omega Y_{i,t}dF(\omega_i) = \int_\Omega \hat{g}(\omega)dF(\omega)K_t^{1-\alpha}(Z_tL_t)^\alpha = mK_t^{1-\alpha}(Z_tL_t)^\alpha.$$

Individual producer profit is

$$\pi_i = \hat{g}(\omega_i)K^{1-\alpha}(ZL)^\alpha - \sum_j w^jL^j - A.$$

Again using the definition of average productivity m, the default cutoff at $\pi_i = 0$ is then

$$\omega^* = \frac{\sum_j w^jL^j + A}{m/\mu_\omega K^{1-\alpha}(ZL)^\alpha}.$$

A.2 Borrower-entrepreneur problem

A.2.1 Preliminaries

We start by defining some preliminaries.

$$\begin{aligned}
Z_A(\omega_t^*) &= 1 - F_{\omega,t}(\omega_t^*)\\
Z_K(\omega_t^*) &= \int_{\omega_t^*}^\infty \hat{g}(\omega)dF_{\omega,t}(\omega) = \frac{m_t}{\mu_\omega}\int_{\omega_t^*}^\infty \omega dF_{\omega,t}(\omega) = \frac{m_t}{\mu_\omega}\omega_t^+
\end{aligned}$$

and $F_{\omega,t}(\cdot)$ is the CDF of $\omega_{i,t}$ with parameters $(\mu_\omega, \sigma^2_{\omega,t})$, where we have defined the integral

$$\omega_t^+ = \int_{\omega_t^*}^\infty \omega dF_{\omega,t}(\omega) = (1 - F_{\omega,t}(\omega_t^*))\mu_\omega + \sigma_{\omega,t}\phi\left(\frac{\omega_t^* - \mu_\omega}{\sigma_{\omega,t}}\right),$$

where $\phi()$ is the pdf of a standard normal random variable. It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$\begin{aligned}
\frac{\partial Z_K(\omega_t^*)}{\partial\omega_t^*} &= \frac{m_t}{\mu_\omega}\frac{\partial}{\partial\omega_t^*}\int_{\omega_t^*}^\infty \omega f_\omega(\omega)d\omega = -\omega_t^* f_\omega(\omega_t^*)\frac{m_t}{\mu_\omega},\\
\frac{\partial Z_A(\omega_t^*)}{\partial\omega_t^*} &= \frac{\partial}{\partial\omega_t^*}\int_{\omega_t^*}^\infty f_\omega(\omega)d\omega = -f_\omega(\omega_t^*),
\end{aligned}$$

where $f_\omega(\cdot)$ is the p.d.f. of a normal distribution with parameters $(\mu_\omega, \sigma_\omega^2)$.

Capital Adjustment Cost Let

$$\Psi(X_t, K_t^B) = \frac{\psi}{2}\left(\frac{X_t}{K_t^B} - (\mu_G + \delta_K)\right)^2 K_t^B.$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) = \psi\left(\frac{X_t}{K_t^B} - (\mu_G + \delta_K)\right) \tag{21}$$

$$\Psi_K(X_t, K_t^B) = -\frac{\psi}{2}\left(\left(\frac{X_t}{K_t^B}\right)^2 - (\mu_G + \delta_K)^2\right) \tag{22}$$

A.2.2 Optimization Problem

We consider the producers's problem in the current period after aggregate and idiosyncratic productivity shocks have been realized, after the intermediary has chosen a default policy, and after the intermediary's random utility penalty is realized. To ensure stationarity of the producer's problem we define the following transformed variables,

$$\left\{\hat{C}_t^B, \hat{X}_t, \hat{A}_t^B, \hat{K}_t^B, \hat{w}_t^j, \hat{G}_t^{T,B}\right\},$$

where for any variable $\hat{var}_t$ denotes division by the current realization of productivity Z_t:

$$\hat{var}_t = \frac{var_t}{Z_t}.$$

For the choices of capital and debt for the next period we further define

$$\hat{\hat{K}}_{t+1}^B = \frac{K_{t+1}^B}{Z_t}$$

and

$$\hat{\hat{A}}_{t+1}^B = \frac{A_{t+1}^B}{Z_t}.$$

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, \hat{W}_t^I, \hat{W}_t^S, \hat{B}_{t-1}^G\right)$ represent state variables exogenous to the borrower-entrepreneur's decision.

Then the stationary problem is

$$\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) = \max_{\{\hat{C}_t^B, \hat{\hat{K}}_{t+1}^B, m_t, \hat{X}_t, \hat{\hat{A}}_{t+1}^B, L_t^j\}} \left\{(1-\beta_B)\left(u_t^B(\hat{C}_t^B, m_t)\right)^{1-1/\nu} + \right.$$
$$\left. + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^B(e^{-g_{t+1}}\hat{\hat{K}}_{t+1}^B, e^{-g_{t+1}}\hat{\hat{A}}_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$
\begin{aligned}
\hat{C}_t^B = &(1-\tau_\Pi^I)Z_K(\omega_t^*)(\hat{K}_t^B)^{1-\alpha}L_t^\alpha + (1-\tau_t^B)\hat{w}_t^B\bar{L}^B + \hat{G}_t^{T,B} + p_t[\hat{X}_t + Z_A(\omega_t^*)(1-(1-\tau_\Pi^B)\delta_K)\hat{K}_t^B] \\
&+ q_t^m\hat{\hat{A}}_{t+1}^B - Z_A(\omega_t^*)\hat{A}_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) \\
&- p_t\hat{\hat{K}}_{t+1}^B - \hat{X}_t - \Psi(\hat{X}_t,\hat{K}_t^B) - (1-\tau_\Pi^I)Z_A(\omega_t^*)\sum_{j=B,I,S}\hat{w}_t^jL_t^j
\end{aligned}
\tag{23}
$$

$$
F\hat{\hat{A}}_t^B \leq \Phi p_t Z_A(\omega_t^*)\hat{K}_t^B \tag{24}
$$

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$
\begin{aligned}
\tilde{\hat{V}}^B\left(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B\right) =&F_\rho(\rho_t^*)\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B)\,|\,\rho<\rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B)\,|\,\rho>\rho_t^*\right] \\
=&F_\rho(\rho_t^*)\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^S(\rho_t<\rho_t^*)) + (1-F_\rho(\rho_t^*))\,\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^S(\rho_t>\rho_t^*)),
\end{aligned}
\tag{25}
$$

where (25) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$
\begin{aligned}
\hat{V}_t^B &\equiv \hat{V}(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B), \\
\hat{V}_{A,t}^B &\equiv \frac{\partial\hat{V}(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B)}{\partial\hat{A}_t^B}, \\
\hat{V}_{K,t}^B &\equiv \frac{\partial\hat{V}(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B)}{\partial\hat{K}_t^B}.
\end{aligned}
$$

Therefore the marginal values of borrowing and of capital of $\tilde{V}^B(\cdot)$ are:

$$
\begin{aligned}
\tilde{\hat{V}}_{A,t}^B &= F_\rho(\rho_t^*)\frac{\partial\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B(\rho_t<\rho_t^*))}{\partial\hat{A}_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B(\rho_t>\rho_t^*))}{\partial\hat{A}_t^B} \\
\tilde{\hat{V}}_{K,t}^B &= F_\rho(\rho_t^*)\frac{\partial\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B(\rho_t<\rho_t^*))}{\partial\hat{K}_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial\hat{V}^B(\hat{K}_t^B,\hat{A}_t^B,\mathcal{S}_t^B(\rho_t>\rho_t^*))}{\partial\hat{K}_t^B}
\end{aligned}
$$

Denote the certainty equivalent of future utility as:

$$
CE_t^B = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^B(\hat{K}_{t+1}^B,\hat{A}_{t+1}^B,\mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}.
$$

Marginal Cost of Default Before deriving optimality conditions, it is useful to compute the marginal consumption loss due to an increased default threshold ω_t^*

$$\begin{aligned}
\frac{\partial C_t^B}{\partial \omega_t^*} =& \frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*}(1-\tau_\Pi^B)(\hat{K}_t^B)^{1-\alpha}L_t^\alpha \\
&+ \frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*}\left[(1-(1-\tau_\Pi^B)\delta_K)p_t\hat{K}_t^B - \hat{A}_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) - (1-\tau_\Pi^B)\sum_j \hat{w}_t^j L_t^j\right] \\
=& -f_\omega(\omega_t^*)\hat{Y}_t \underbrace{\left[(1-\tau_\Pi^B)\omega_t^* + \frac{(1-(1-\tau_\Pi^B)\delta_K)p_t\hat{K}_t^B - \hat{A}_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) - (1-\tau_\Pi^B)\sum_j \hat{w}_t^j L_t^j}{\hat{Y}_t}\right]}_{=\mathcal{F}(\hat{K}_t^B, \hat{A}_t^B, p_t, q_t^m, \hat{w}_t^j)=\mathcal{F}_t} \\
=& -f_\omega(\omega_t^*)\hat{Y}_t\mathcal{F}_t,
\end{aligned}$$

where we further defined aggregate output, scaled by μ_ω,

$$\hat{Y}_t = \frac{m_t}{\mu_\omega}(\hat{K}_t^B)^{1-\alpha}L_t^\alpha.$$

The function $\mathcal{F}(\hat{K}_t^B, \hat{A}_t^B, p_t, q_t^m, \hat{w}_t^j)$ has an intuitive interpretation as the marginal loss, expressed in consumption units per unit of aggregate output, to producers from an increase in the default threshold. The first term is the loss in output and the second term is the loss of capital due to defaulting members. The second and the third term represent gains due to debt erased in foreclosure and labor costs of bankrupt firms.

A.2.3 First-order conditions

Loans The FOC for loans $\hat{\hat{A}}_{t+1}^B$ is:

$$\begin{aligned}
&q_t^m \frac{(\hat{u}_t^B)^{1-1/\nu}}{\hat{C}_t^B}(1-\beta_B)(\hat{V}_t^B)^{1/\nu} = \\
&\lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^B)^{-\sigma_B}\tilde{\hat{V}}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}
\end{aligned} \tag{26}$$

where λ_t^B is the Lagrange multiplier on the constraint in (24).

Capital Similarly, the FOC for new capital $\hat{\hat{K}}_{t+1}^B$ is:

$$\begin{aligned}
&p_t \frac{(1-\beta_B)(\hat{V}_t^B)^{1/\nu}(\hat{u}_t^B)^{1-1/\nu}}{\hat{C}_t^B} = \\
&\beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^B)^{-\sigma_B}\tilde{\hat{V}}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(\hat{V}_t^B)^{1/\nu}
\end{aligned} \tag{27}$$

Investment The FOC for investment $\hat{X}_t$ is:

$$[1 + \Psi_X(\hat{X}_t^B, \hat{K}_t^B) - p_t] \frac{(1-\beta_B)(\hat{U}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B} = 0,$$

which simplifies to

$$1 + \Psi_X(\hat{X}_t^B, \hat{K}_t^B) = p_t.$$

Labor Inputs Defining $\gamma_B = 1 - \gamma_I - \gamma_S$, aggregate labor input is

$$L_t = \prod_{j=B,I,S} (L_t^j)^{\gamma_j}.$$

We further compute

$$\frac{\partial \omega_t^*}{\partial L_t^j} = \frac{\hat{w}_t^j}{\hat{Y}_t} - \omega_t^* \frac{m_t}{\mu_\omega} \frac{\mathrm{MPL}_t^j}{\hat{Y}_t},$$

defining the marginal product of labor of type j as

$$\mathrm{MPL}_t^j = \alpha \gamma_j \frac{L_t}{L_t^j} \left(\frac{\hat{K}_t^B}{L_t} \right)^{1-\alpha}.$$

The FOC for labor input L_t^j is then

$$\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B} \left[(1-\tau_\Pi^B) Z_K(\omega_t^*) \mathrm{MPL}_t^j - (1-\tau_\Pi^B) Z_A(\omega_t^*) \hat{w}_t^j + \frac{\partial \omega_t^*}{\partial L_t^j} \frac{\partial \hat{C}_t^B}{\partial \omega_t^*} \right] = 0,$$

which yields

$$(1-\tau_\Pi^B) Z_K(\omega_t^*) \mathrm{MPL}_t^j = (1-\tau_\Pi^B) Z_A(\omega_t^*) \hat{w}_t^j + f_\omega(\omega_t^*) \left(\hat{w}_t^j - \omega_t^* \frac{m_t}{\mu_\omega} \mathrm{MPL}_t^j \right) \mathcal{F}_t. \qquad (28)$$

Aggregate Productivity (Effort) First, compute the derivatives

$$\frac{\partial Z_K(\omega_t^*)}{\partial m_t} = \frac{\omega_t^+}{\mu_\omega} + \frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial m_t},$$

$$\frac{\partial \omega_t^*}{\partial m_t} = -\frac{1}{\mu_\omega} (\hat{K}_t^B)^{1-\alpha} L_t^\alpha \frac{\omega_t^*}{\hat{Y}_t} = -\frac{\hat{Y}_t}{m_t} \frac{\omega_t^*}{\hat{Y}_t}.$$

Further, we compute

$$u_{t,m}^B(\hat{C}_t^B, m_t) = -\frac{1}{1-\phi} \left(\frac{m_t}{\mu_\omega} \right)^{\frac{\phi}{1-\phi}} \left[\eta + \sigma_{\omega,t}^2 \left(\frac{\eta}{1-\phi} \right)^2 \left(\frac{m_t}{\mu_\omega} \right)^{\frac{1}{1-\phi}} \right] u_t^B.$$

Then the FOC for m_t is

$$0 = (1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}\left[\frac{\hat{u}_m(\hat{C}_t^B, m_t)}{\hat{u}_t^B} + \frac{1}{\hat{C}_t^B}\left((1-\tau_\Pi^B)\frac{\hat{Y}_t}{m_t}\omega_t^+ + \frac{\partial\omega_t^*}{\partial m_t}\frac{\partial\hat{C}_t^B}{\partial\omega_t^*}\right)\right],$$

which can be written as

$$\frac{m_t\hat{C}_t^B}{1-\phi}\left(\frac{m_t}{\mu_\omega}\right)^{\frac{\phi}{1-\phi}}\left[\eta + \sigma_{\omega,t}^2\left(\frac{\eta}{1-\phi}\right)^2\left(\frac{m_t}{\mu_\omega}\right)^{\frac{1}{1-\phi}}\right] = \hat{Y}_t\left((1-\tau_\Pi^B)\omega_t^+ + f_\omega(\omega_t^*)\omega_t^*\mathcal{F}_t\right). \tag{29}$$

A.2.4 Marginal Values of State Variables and SDF

Loans Taking the derivative of the value function with respect to $\hat{A}_t^B$ gives:

$$\begin{aligned}\hat{V}_{A,t}^B &= \left[-\left(1-(1-\theta)\tau_\Pi^I + \delta q_t^m\right)Z_A(\omega_t^*) + \frac{\partial\omega_t^*}{\partial\hat{A}_t^B}\frac{\partial\hat{C}_t^B}{\partial\omega_t^*}\right]\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B} \\ &= -\left[\left(1-(1-\theta)\tau_\Pi^I + \delta q_t^m\right)Z_A(\omega_t^*) + f_\omega(\omega_t^*)\mathcal{F}_t\right]\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B},\end{aligned} \tag{30}$$

where we used the fact that $\frac{\partial\omega_t^*}{\partial\hat{A}_t^B} = \frac{1}{\hat{Y}_t}$.

Capital Taking the derivative of the value function with respect to $\hat{K}_t^B$ gives:

$$\begin{aligned}\hat{V}_{K,t}^B = &\left[p_t Z_A(\omega_t^*)\left(1-(1-\tau_\Pi^B)\delta_K\right) + (1-\tau_\Pi^B)(1-\alpha)Z_K(\omega_t^*)\left(\frac{\hat{K}_t^B}{L_t}\right)^{-\alpha} - \Psi_K(\hat{X}_t^B, \hat{K}_t^B) + \frac{\partial\omega_t^*}{\partial\hat{K}_t^B}\frac{\partial\hat{C}_t^B}{\partial\omega_t^*}\right. \\ &\left. + \tilde{\lambda}_t^B\Phi p_t\left(Z_A(\omega_t^*) + \hat{K}_t^B\frac{\partial Z_A(\omega_t^*)}{\partial\omega_t^*}\frac{\partial\omega_t^*}{\partial\hat{K}_t^B}\right)\right]\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B}.\end{aligned}$$

Taking the derivative

$$\frac{\partial\omega_t^*}{\partial\hat{K}_t^B} = -\frac{m_t}{\mu_\omega}\frac{\omega_t^*}{\hat{Y}_t}(1-\alpha)\left(\frac{\hat{K}_t^B}{L_t}\right)^{-\alpha},$$

we get

$$\begin{aligned}\hat{V}_{K,t}^B = &\left\{p_t Z_A(\omega_t^*)\left(1-(1-\tau_\Pi^B)\delta_K + \Phi\tilde{\lambda}_t^B\right) + (1-\tau_\Pi^B)(1-\alpha)\left(\frac{\hat{K}_t^B}{L_t}\right)^{-\alpha} - \Psi_K(\hat{X}_t^B, \hat{K}_t^B)\right. \\ &\left. +(1-\alpha)f_\omega(\omega_t^*)\omega_t^*\left[\frac{m_t}{\mu_\omega}\left(\frac{\hat{K}_t^B}{L_t}\right)^{-\alpha}\mathcal{F}_t + \tilde{\lambda}_t^B\Phi p_t\right]\right\}\frac{(1-\beta_B)(\hat{u}_t^B)^{1-1/\nu}(\hat{V}_t^B)^{1/\nu}}{\hat{C}_t^B}.\end{aligned} \tag{31}$$

SDF We can define the stochastic discount factor (SDF) from t to $t+1$ of borrowers, conditional on a particular realization of ρ_{t+1} as:

$$\mathcal{M}^B_{t,t+1}(\rho_{t+1}) = \beta_B e^{-\sigma_B g_{t+1}} \left(\frac{\hat{u}^B_{t+1}(\rho_{t+1})}{\hat{u}^B_t}\right)^{1-1/\nu} \left(\frac{\hat{C}^B_{t+1}(\rho_{t+1})}{\hat{C}^B_t}\right)^{-1} \left(\frac{\hat{V}^B_{t+1}(\rho_{t+1})}{CE^B_t}\right)^{1/\nu} \left(\frac{\tilde{\hat{V}}^B_{t+1}}{CE^B_t}\right)^{-\sigma_B} \tag{32}$$

Conditional on information at time t, every endogenous random variable at $t+1$ is adapted both to productivity growth g_{t+1} and to the event $\mathbb{1}[\rho_{t+1} < \rho^*_{t+1}]$. For any such variable we define the sum

$$\vec{x}_{t+1} = \mathbb{1}[\rho_{t+1} < \rho^*_{t+1}] \left(x_{t+1} \,|\, \rho_{t+1} < \rho^*_{t+1}\right) + \mathbb{1}[\rho_{t+1} \geq \rho^*_{t+1}] \left(x_{t+1} \,|\, \rho_{t+1} \geq \rho^*_{t+1}\right).$$

Hence for any payoff x_{t+1} we can write the appropriately discounted payoff as

$$\vec{\mathcal{M}}^B_{t,t+1} \vec{x}_{t+1}.$$

A.2.5 Euler Equations

Loans Recall that $\tilde{V}^B_{A,t+1}$ is a linear combination of $V^B_{A,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{A,t+1}$ given by equation (30). Substituting in for $\tilde{V}^B_{A,t+1}$ in (26) and using the SDF expression, we get the recursion:

$$q^m_t = \tilde{\lambda}^B_t F + \mathrm{E}_t \left\{ \vec{\mathcal{M}}^B_{t,t+1} \left[Z_A(\vec{\omega}^*_{t+1}) \left(1 - (1-\theta)\tau_\Pi + \delta \vec{q}^m_{t+1}\right) + f_\omega(\vec{\omega}^*_{t+1}) \vec{\mathcal{F}}_{t+1} \right] \right\}. \tag{33}$$

Capital Likewise, recall that $\tilde{V}^B_{K,t+1}$ is a linear combination of $V^B_{K,t+1}$ conditional on ρ_t being below and above the threshold, and with each $V^B_{K,t+1}$ given by equation (31). Substituting in for $\tilde{V}^B_{K,t+1}$ and using the SDF expression, we get the recursion:

$$\begin{aligned} p_t = \mathrm{E}_t \Bigg[\vec{\mathcal{M}}^B_{t,t+1} \Bigg\{ & \vec{p}_{t+1} Z_A(\vec{\omega}^*_{t+1}) \left(1 - (1-\tau_\Pi)\delta_K + \Phi \vec{\tilde{\lambda}}^B_{t+1}\right) + (1-\tau_\Pi)(1-\alpha) Z_K(\vec{\omega}^*_{t+1}) \left(\frac{\vec{\hat{K}}^B_{t+1}}{\vec{L}_{t+1}}\right)^{-\alpha} \\ & - \Psi_K(\vec{\hat{X}}^B_{t+1}, \vec{\hat{K}}^B_{t+1}) + (1-\alpha) f_\omega(\vec{\omega}^*_{t+1}) \vec{\omega}^*_{t+1} \left(\frac{\vec{m}_{t+1}}{\mu_\omega} \left(\frac{\vec{\hat{K}}^B_{t+1}}{\vec{L}_{t+1}}\right)^{-\alpha} \vec{\mathcal{F}}_{t+1} + \Phi \vec{\tilde{\lambda}}^B_{t+1} \vec{p}_{t+1} \right) \Bigg\} \Bigg]. \end{aligned} \tag{34}$$

A.3 Intermediaries

A.3.1 Statement of stationary problem

As for borrower-entrepreneurs, we define the following transformed variables for intermediaries:

$$\{\hat{W}^I_t, \hat{C}^I_t, \hat{A}^I_{t+1}, \hat{G}^{T,I}_t, \hat{B}^I_t\}.$$

Denote by $\hat{W}^I_t$ risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{\hat{W}}^I_t = (1 - D(\rho_t))\hat{W}^I_t,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}^I_t = \left(g_t, \sigma_{\omega,t}, \hat{K}^B_t, \hat{A}^B_t, \hat{B}^G_{t-1}, \hat{W}^S_t\right)$ denote all other aggregate state variables exogenous to intermediaries.

After the default decision, intermediaries face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\hat{V}^I(\tilde{\hat{W}}^I_t, \tilde{\rho}_t, \mathcal{S}^I_t) = \max_{\hat{C}^I_t, \hat{A}^I_{t+1}, \hat{B}^I_t} \left\{ (1-\beta_I)\left[\frac{\hat{C}^I_t}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu} + \beta_I \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^I\left(\hat{W}^I_{t+1}, \mathcal{S}^I_{t+1}\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}} \right\}^{\frac{1}{1-1/\nu}} \tag{35}$$

subject to:

$$(1-\tau^I)\hat{w}^I_t \bar{L}^I + \tilde{\hat{W}}^I_t + \hat{G}^{T,I}_t = \hat{C}^I_t + q^m_t \hat{A}^I_{t+1} + (q^f_t + \tau^\Pi_I r^f_t - \kappa I_{\{\hat{B}^I_t<0\}})\hat{B}^I_t, \tag{36}$$

$$\hat{W}^I_{t+1} = e^{-g_{t+1}}\left[\left(\tilde{M}_{t+1} + Z_A(\omega^*_{t+1})\delta q^m_{t+1}\right)\hat{A}^I_{t+1} + \hat{B}^I_t\right], \tag{37}$$

$$\hat{B}^I_t \geq -\xi q^m_t \hat{A}^I_{t+1}, \tag{38}$$

$$\hat{A}^R_{t+1} \geq 0, \tag{39}$$

$$\mathcal{S}^I_{t+1} = h(\mathcal{S}^I_t). \tag{40}$$

For the evolution of intermediary wealth in (37), we have defined the total after-tax payoff per bond

$$\tilde{M}_{t+1} = (1-(1-\theta)\tau^\Pi_I)Z_A(\omega^*_{t+1}) + \hat{M}_{t+1}/A^B_{t+1},$$

where $\hat{M}_{t+1}$ is the total recovery value of bankrupt borrower firms seized by intermediaries, as defined in (14).

The continuation value $\tilde{\hat{V}}^I\left(\hat{W}^I_{t+1}, \mathcal{S}^I_{t+1}\right)$ is the outcome of the optimization problem intermediaries face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{\hat{V}}^I(\hat{W}^I_t, \mathcal{S}^I_t) = \max_{D(\rho)} \mathrm{E}_\rho\left[D(\rho)\hat{V}^I(0, \rho, \mathcal{S}^I_t) + (1-D(\rho))\hat{V}^I(\hat{W}^I_t, 0, \mathcal{S}^I_t)\right] \tag{41}$$

Define the certainty equivalent of future utility as:

$$CE^I_t = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^I\left(\hat{W}^I_{t+1}, \mathcal{S}^I_{t+1}\right)\right)^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}. \tag{42}$$

A.3.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0,1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $\hat{V}^I(W, \rho, \mathcal{S}_t^R)$ defined in (35) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given $\hat{W}_t^I$, intermediaries optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (41) as

$$\begin{aligned}\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho \left[\mathbb{1}[\rho < \rho^*]\, \hat{V}^I(0, \rho, \mathcal{S}_t^I) + (1 - \mathbb{1}[\rho < \rho^*])\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\, \mathrm{E}_\rho \left[\hat{V}^I(0, \rho, \mathcal{S}_t^I) \,|\, \rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$\hat{V}^I(0, \rho_t^*, \mathcal{S}_t^I) = \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $\hat{V}^I(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = \hat{V}^I(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)), \tag{43}$$

and by substituting the solution into (41), we obtain

$$\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^I) = F_\rho(\rho_t^*)\mathrm{E}_\rho \left[\hat{V}^I(0, \rho, \mathcal{S}_t^I) \,|\, \rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I). \tag{44}$$

Equations (35), (43), and (44) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (43) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1 - \beta_I)\hat{C}_t^I) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_I(CE_t^I)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1 - \beta_I)\log(\hat{C}_t^I) + \beta_I\log(CE_t^I) - \log(x) - (1 - \beta_I) & \text{if } \nu = 1. \end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned}&(q_t^f + \tau^\Pi r_t^f - \kappa I_{\{\hat{B}_t^I < 0\}})\frac{(1 - \beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \\ &\lambda_t^I + \beta_I \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^I)^{-\sigma_I}\tilde{\hat{V}}_{W,t+1}^I](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu}\end{aligned} \tag{45}$$

where λ_t^I is the Lagrange multiplier on the borrowing constraint (38).

The first order condition for loans is:

$$
\begin{aligned}
&(q_t^m + \tau^\Pi r_t^m F)\frac{(1-\beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \lambda_t^I \xi q_t^m + \mu_t^I \\
&+ \beta_I \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^I)^{-\sigma_I}\tilde{\hat{V}}_{W,t+1}^I\left(\tilde{M}_{t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu}, \qquad (46)
\end{aligned}
$$

where μ_t^I is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (39).

A.3.3 Marginal value of wealth and SDF

Differentiating (44) gives the marginal value of wealth

$$
\tilde{\hat{V}}_{W,t}^I = (1 - F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)}{\partial \hat{W}_t^I},
$$

where

$$
\frac{\partial \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)}{\partial \hat{W}_t^I} = (\hat{C}_t^I)^{-1/\nu}(1-\beta_I)(\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I))^{1/\nu},
$$

The stochastic discount factor of intermediaries conditional on $\rho_{t+1} \geq \rho_{t+1}^*$ is therefore

$$
\mathcal{M}_{t,t+1}^I = \beta_I e^{-\sigma_I g_{t+1}}\left(\frac{\hat{V}^I(\hat{W}_{t+1}^I, 0, \mathcal{S}_{t+1}^I)}{CE_t^I}\right)^{1/\nu}\left(\frac{\tilde{\hat{V}}^I(\tilde{\hat{W}}_{t+1}^I, \mathcal{S}_{t+1}^I)}{CE_t^I}\right)^{-\sigma_I}\left(\frac{\hat{C}_{t+1}^I}{\hat{C}_t^I}\right)^{-1/\nu}.
$$

A.3.4 Euler Equations

It is then possible to show that the FOC with respect to $\hat{B}_t^I$ and $\hat{A}_{t+1}^I$, respectively, are:

$$
q_t^f = \tilde{\lambda}_t^I + \mathrm{E}_t\left[(1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^I\right] + \kappa I_{\{\hat{B}_t^I < 0\}} - \tau^\Pi r_t^f, \qquad (47)
$$

$$
q_t^m(1 - \xi\tilde{\lambda}_t^I) = \tilde{\mu}_t^I + \mathrm{E}_t\left[(1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^I\left(\tilde{M}_{t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)\right]. \qquad (48)
$$

A.4 Savers

A.4.1 Statement of stationary problem

For savers, we define the following transformed variables:

$$
\{\hat{W}_t^S, \hat{C}_t^S, \hat{B}_t^S, \hat{G}_t^{T,S}\}.
$$

Let $\mathcal{S}_t^S = \left(g_t, \sigma_{\omega,t}, \hat{K}_t^B, \hat{A}_t^B, \hat{W}_t^I, \hat{B}_{t-1}^G\right)$ be the saver's state vector capturing all exogenous state variables. Scaling by productivity, the stationary problem of the saver – after the intermediary has made default her decision and the utility cost of default is realized – is:

$$\hat{V}^S(\hat{W}_t^S, \mathcal{S}_t^S) = \max_{\{\hat{C}_t^S, \hat{B}_t^S\}} \left\{ (1-\beta_S)\left[\hat{C}_t^S\right]^{1-1/\nu} + \beta_S \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^S(\hat{W}_{t+1}^S, \mathcal{S}_{t+1}^S) \right)^{1-\sigma_S} \right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$\hat{C}_t^S = (1-\tau_t^S)\hat{w}_t^S \bar{L}^S + \hat{G}_t^{T,S} + \hat{W}_t^S - q_t^f \hat{B}_t^S \tag{49}$$
$$\hat{W}_{t+1}^S = e^{-g_{t+1}} \hat{B}_t^S \tag{50}$$
$$\hat{B}_t^S \geq 0 \tag{51}$$
$$\mathcal{S}_{t+1}^S = h(\mathcal{S}_t^S) \tag{52}$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$\hat{V}_t^S \equiv \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S),$$
$$\hat{V}_{W,t}^S \equiv \frac{\partial \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S)}{\partial \hat{W}_t^S},$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^S(\hat{W}_t^S, \mathcal{S}_t^S) \right)^{1-\sigma_S} \right].$$

Like borrower-entrepreneurs, savers must take into account the intermediary's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{\hat{V}}_{W,t}^S = F_\rho(\rho_t^*) \frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t < \rho_t^*))}{\partial \hat{W}_t^S} + (1 - F_\rho(\rho_t^*)) \frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t > \rho_t^*)}{\partial \hat{W}_t^S}.$$

A.4.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f (\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu} = \lambda_t^S + \beta_S \mathrm{E}_t[(e^{g_{t+1}} \tilde{\hat{V}}_{t+1}^S)^{-\sigma_S} \tilde{\hat{V}}_{W,t+1}^S](CE_t^S)^{\sigma_S - 1/\nu}(\hat{V}_t^S)^{1/\nu} \tag{53}$$

where λ_t^S is the Lagrange multiplier on the no-borrowing constraint (51).

A.4.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$\hat{V}_{W,t}^S = (\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu}, \tag{54}$$

and for the continuation value function:

$$\tilde{\hat{V}}^S_{W,t} = F_\rho(\rho^*_t)\frac{\partial V^S(\hat{W}^S_t, \mathcal{S}^S_t(\rho_t < \rho^*_t))}{\partial \hat{W}^S_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial \hat{V}^S(\hat{W}^S_t, \mathcal{S}^S_t(\rho_t > \rho^*_t)}{\partial \hat{W}^S_t}.$$

Defining the SDF in the same fashion as we did for borrowers, we get:

$$\mathcal{M}^S_{t,t+1}(\rho_{t+1}) = \beta_S e^{-\sigma_S g_{t+1}} \left(\frac{\hat{V}^S_{t+1}(\rho_{t+1})}{CE^S_t}\right)^{1/\nu} \left(\frac{\tilde{\hat{V}}^S_{t+1}}{CE^S_t}\right)^{-\sigma_S} \left(\frac{\hat{C}^S_{t+1}(\rho_{t+1})}{\hat{C}^S_t}\right)^{-1/\nu},$$

and

$$\vec{\mathcal{M}}^S_{t,t+1} = \mathbb{1}[\rho_{t+1} < \rho^*_{t+1}] \left(\mathcal{M}^S_{t,t+1} \,|\, \rho_{t+1} < \rho^*_{t+1}\right) + \mathbb{1}[\rho_{t+1} \geq \rho^*_{t+1}] \left(\mathcal{M}^S_{t,t+1} \,|\, \rho_{t+1} \geq \rho^*_{t+1}\right).$$

A.4.4 Euler Equations

Combining the first-order condition for short-term bonds (53) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q^f_t = \tilde{\lambda}^S_t + \mathrm{E}_t\left[\vec{\mathcal{M}}^S_{t,t+1}\right] \tag{55}$$

where $\tilde{\lambda}^S_t$ is the original multiplier λ^S_t divided by the marginal value of wealth.

B Calibration Appendix

B.1 Long-term corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by $F = \frac{\theta}{1-\delta}$, a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for δ and θ.

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices[26] we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices[27] we obtain a time

[26] Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively

[27] They are named C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively.

series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields r_t. Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter δ as

$$P^G(r_t) = \frac{1}{1+r_t}\left[1 + \delta P^G(r_t)\right]$$

Solving for $P^G(y_t)$, we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units X of the geometric bond with parameter δ one needs to sell to hedge one unit of plain vanilla bond P^c against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta,X} \sum_{t=1997.1}^{2015.12} \left[P^c(r_t) - XP^G(r_t;\delta)\right]^2$$

We estimate $\delta = 0.937$ and $X = 12.9$, yielding an average pricing error of only 0.41%. This value for δ implies a time series of durations $D_t = -\frac{1}{P_t^G}\frac{dP_t^G}{dr_t}$ with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it D_t years from now. The principal of this loan is just the price of the corresponding D_t maturity zero-coupon bond $\frac{1}{(1+r_t)^{D_t}}$

We set the "principal" F of one unit of the geometric bond to be some fraction θ of the undiscounted sum of all its cash flows $\frac{\theta}{1-\delta}$, where

$$\theta = \frac{1}{N}\sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get $\theta = 0.582$ and $F = 9.18$.