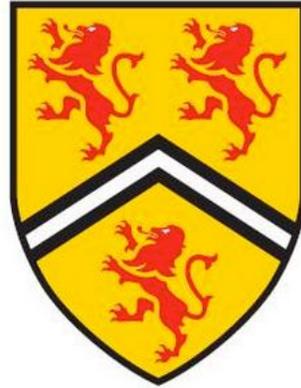


# University of Waterloo



Faculty of Engineering  
Department of Mechanical & Mechatronics Engineering

## **Laboratory #1: Introduction to Control Engineering**

MTE - 360 Automatic Control Systems

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Group # 13

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# 1 – Parameter Identification through Step Response Measurement

## 1b. Determining Velocity Gain $K_v$ and Time Constant $\tau_v$

The sample that was examined is the occurrence between 2.5 and 3 seconds. The maximum and minimum velocity values of the response in the sample window were recorded to be 186.368 mm/s and -163.072 mm/s. The amplitude is calculated to be 349.44 mm/s and the value for  $K_v$  can be determined:

$$186.368 \text{ mm/s} - (-163.072 \text{ mm/s}) = 349.44 \text{ mm/s} \div 3V = \mathbf{116.48 \text{ [(mm/s) / V]}}$$

The amplitude is divided by three to match the input voltage's scale. The time constant is found by taking the velocity amplitude, multiplying it by one time constant (63.21%) and subtracted by the minimum velocity to find the magnitude at the first time constant.

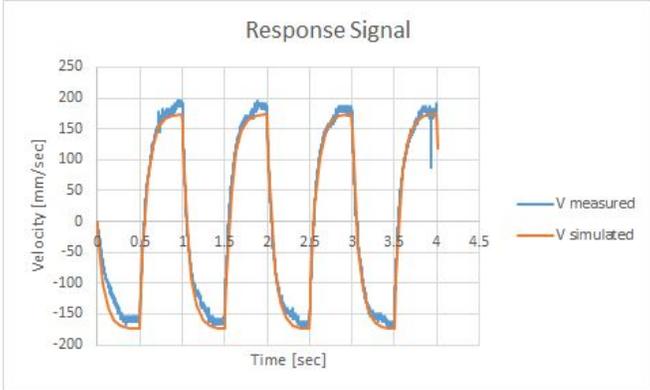
$$349.44 \text{ mm/s} * 0.6321 = 220.881 \text{ mm/s} - 163.072 \text{ mm/s} = 57.809 \text{ mm/s}$$

The time the cart reaches the velocity of 57.809 mm/s is 2.5772 seconds and the time is subtracted by 2.5, the starting sample time, to obtain the time constant,  $\tau_v$ .

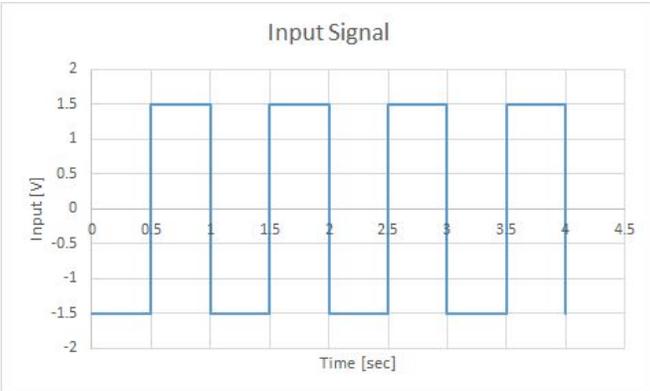
$$2.5772 - 2.5 = \mathbf{0.0772 \text{ [s]}}$$

## 1c. Theoretical Velocity Response vs. Measured Velocity Response

The period and amplitude of the simulated and measured response are very similar as shown in Figure 1. There are slight discrepancies due to the jitter and noise of the system in the measured response. This confirms that the time constant and the calculated gain value selected was accurate due to the similarities in the graph of the measured and simulated response.



**Figure 1:** Simulated Response vs Measured Response

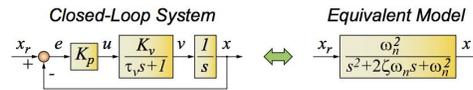


**Figure 2:** Input Signal

The Simulink model used to obtain the theoretical velocity response is shown in the Appendix.

## 2 – Proportional (P-type) Position Control

### 2b. Transfer Function Derivation



**Figure 3:** Equivalent 2nd order model for P-controlled servo system.

Using Figure 3:

$$\frac{x}{x_r} = \frac{G}{1+GH} \text{ due to the feedback loop where } G = \frac{K_p K_v}{s^2 + \frac{s}{\tau_v}} \text{ and } H = 1$$

$$\text{Substituting G and H } \frac{x}{x_r} = \frac{\frac{K_p K_v}{s^2 + \frac{s}{\tau_v}}}{1 + \frac{K_p K_v}{s^2 + \frac{s}{\tau_v}}} = \frac{\frac{K_p K_v}{s^2 + \frac{s}{\tau_v}}}{\frac{s^2 + \frac{s}{\tau_v} + K_p K_v}{s^2 + \frac{s}{\tau_v}}} = \frac{K_p K_v}{s^2 + \frac{s}{\tau_v} + \frac{K_p K_v}{\tau_v}}$$

$$\text{hence } \omega_n^2 = \frac{K_p K_v}{\tau_v}, \omega_n = \sqrt{\frac{K_p K_v}{\tau_v}} \text{ and } \zeta = \frac{1}{\tau_v 2\omega_n}$$

The computed values for natural frequency and damping ratio while varying  $K_p$  are displayed below:

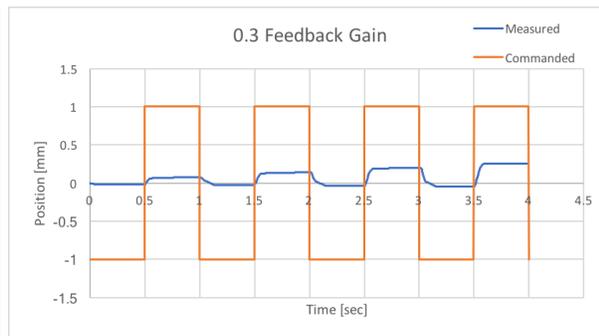
$K_p$	$K_v$	$\tau_v$	$\omega_n$	$\zeta$
0.3	116.48	0.0772	21.275	0.3044
0.6	116.48	0.0772	30.088	0.2153
2.1	116.48	0.0772	56.289	0.1151

**Table 1:** Natural Frequency and Damping Ratio calculations for varying  $K_p$  values

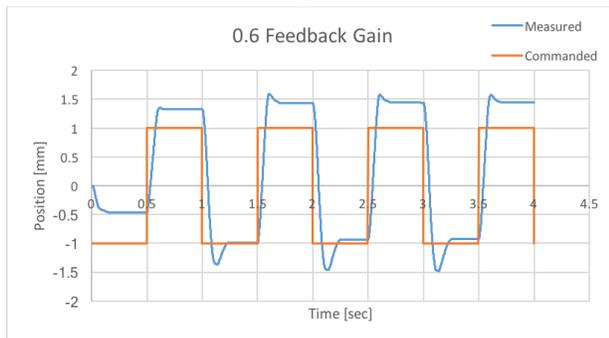
### 2c. Closed-Loop Position Step Response Analysis



**Figure 4:** 0.3 Feedback Gain Control Graph



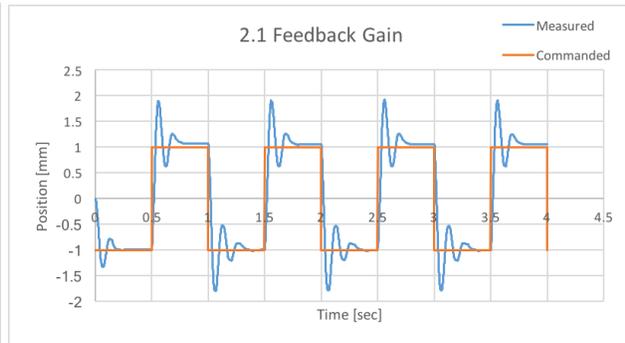
**Figure 5:** 0.3 Feedback Gain Position Graph



**Figure 6: 0.6 Feedback Gain Control Graph**



**Figure 7: 0.6 Feedback Gain**



**Figure 8: 2.1 Feedback Gain Control Graph**

Observations:

**Risetime**

- defined as the time required to reach 100% of steady state value for the first time, (approx  $t_r = \frac{1.8}{\omega_n}$ )
- from the derivation,  $\omega_n^2 = \frac{K_p K_v}{\tau_v}$ ,  $K_p \propto \omega_n$  and  $\omega_n \propto \frac{1}{t_{rise}}$
- therefore as feedback gain increases, rise time decreases
- With feedback gain at 0.3, the measured position never reaches the commanded position and no rise time is observed. When the feedback gain is set to 0.6 the measured value reaches the commanded position fairly quickly, this occurs even faster when the feedback gain is at 2.1.

**Figure 9: 2.1 Feedback Gain**

**Overshoot**

- occurs at the first occurrence of when the derivative is zero or defined by the equation:  $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$ .
- from the derivation,  $K_p \propto \omega_n$  which is also  $\omega_n \propto \frac{1}{\zeta}$
- therefore the higher damping ratio, the lower the overshoot. Thus higher feedback gain creates greater overshoot.
- At a feedback gain of 0.3, there is no overshoot because the measured position does not exceed 100% of the steady state value. The overshoot at feedback gains of 0.6 and 2.1 are approximately 1.6 mm and 1.9 mm, respectively. It is observed that higher feedback gain results in a higher the overshoot.

**Steady State Error**

- defined by the difference between the commanded and measured signals.
- The largest discrepancies occur when the feedback gain is at 0.3. The least error occurs at 0.6 feedback gain however, the cart converges at 1.5 mm instead of 1 mm. This is an error in measurement and is not perfectly representative of how the cart performed. There was either a positive DC offset applied to positive measurements in our measurement stack or a negative DC offset applied to only positive measurements somewhere in the actual control signal loop that caused the cart to consistently overshoot.

**Control Signal**

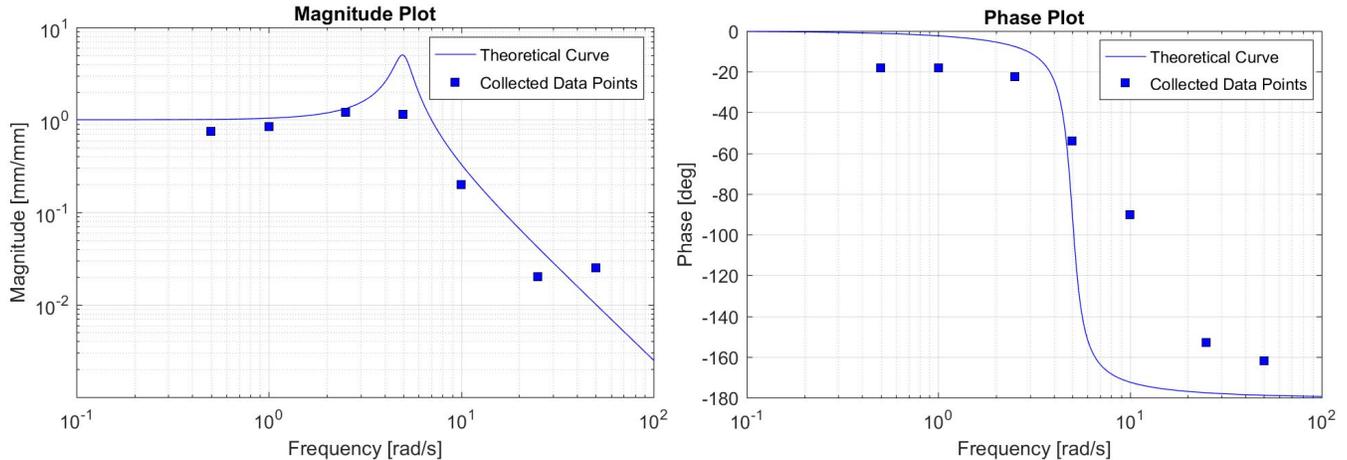
- defined by the signal outputted by the controller into the mechanical drive
- the control signal increases in polynomial order with increase in proportional feedback gain as seen in figures 4, 6, and 8, respectively.
- Observations from 0.3 feedback gain shows that over a long period, the cart is provided enough voltage and will eventually reach the desired position, however, due to the short period the positions change, it does not reach steady state in the given amount of time. The control signals in the 0.6 and 2.1 feedback gain gives the cart enough power to reach the desired positions.

### 3 – Measurement of Closed-loop Frequency Response:

#### 3b. Frequency Response Measurement Summary

Frequency $f$ [Hz]	Input $\Delta x_r$ [mm]	Output $\Delta x$ [mm]	Time Lag $t$ [sec]	Gain $ G $ [mm/mm]	Phase $\angle G$ [deg]
0.5	2	1.5	0.2	0.75	-18
1.0	2	1.7	0.1	0.85	-18
2.5	2	2.4	0.05	1.2	-22.5
5.0	2	2.3	0.06	1.15	-54
10.0	2	0.4	0.05	0.2	-90
25.0	2	0.04	0.034	0.014	-153
50.0	2	0.05	0.018	0.025	-162

#### 3c. Theoretical Frequency Response -- Mistake in the data collected, wn should be at 30s



**Figure 11: Bode Plot of the 0.6 Kp response**

The MATLAB code used to generate Figure 11 is attached in the Appendix. There are various things to note about the features of the above graphs:

- DC gain observed is approximately one looking at the y intercept.
- A resonance magnitude occurs around 4 rad/s in the theoretical curve. This is somewhat observed in the measured data as the two values close to 4 rad/s (2.5 and 5 rad/s) have slightly elevated values of D.C. gain that exceed 1.
- At frequencies above 4 rad/s the magnitude of the response quickly begins to drop off (gain attenuation). This drop off begins starts to level off at the last data point however, which is not what the theoretical curve suggests. This leveling off is likely due to the measurement accuracy of our position sensors, as at higher frequencies the noise and lack of precision in our measurement system will contribute more to the measurement than the actual movement of the cart.
- The theoretical phase angle curve and collected data points also match to a fair extent. The output never quite has 0° of delay from the input at the beginning because our controlling hardware/software will always some delay associated with it that cannot be eliminated. At the higher frequencies, a transition to full 180° is also not quite observed, most likely due to measurement error increasing as the magnitude and frequency of our measurements has increased greatly and may be hitting the limits of our sensing hardware/software.

- closed loop servo performance is better than open loop as it allows for feedback on position to reach the set point

## Appendix

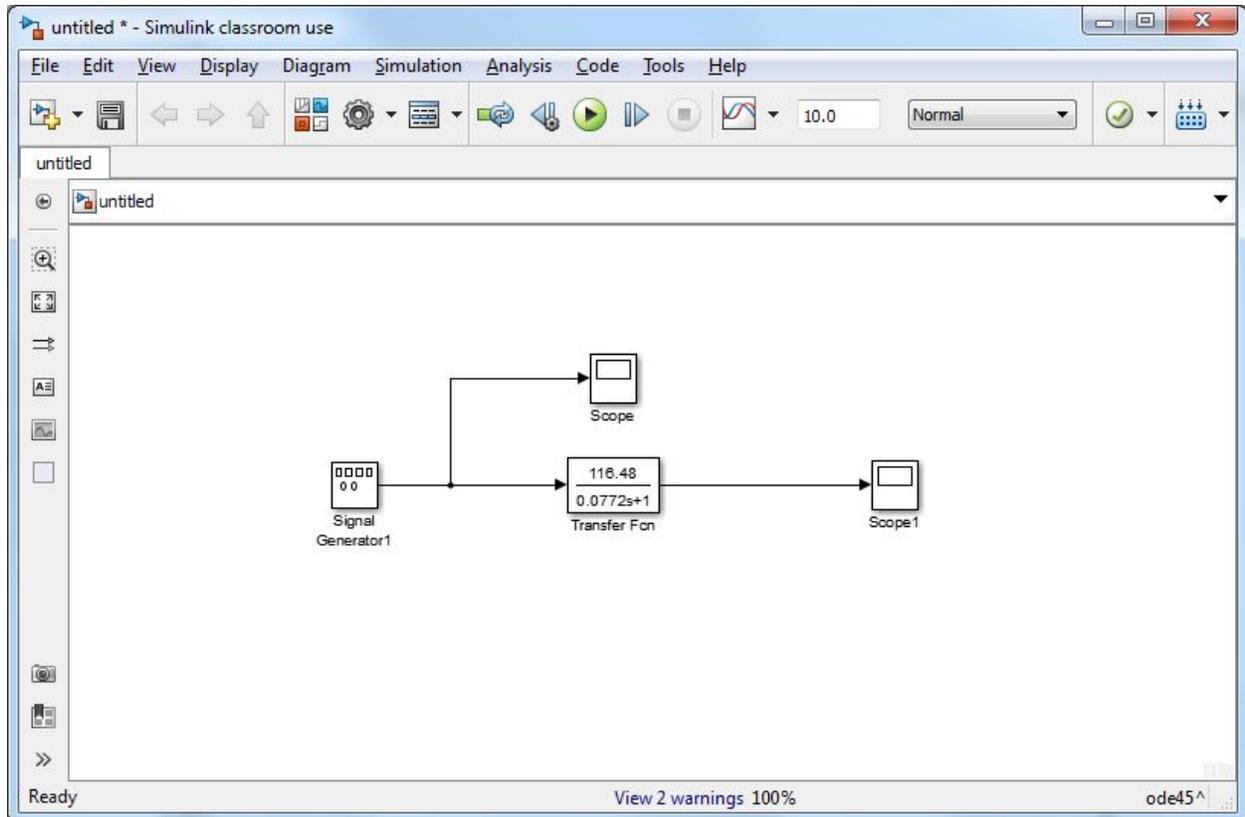


Figure 12: Simulink model of velocity response

### Part 3c. Code:

```
clear all;
close all;
clc;

% define 2nd order transfer function for Kp 0.6%
wn = 30.088; % natural frequency [rad/s]
zeta = 0.2153; % damping ratio [ ]
G = tf([wn*wn],[1 2*zeta*wn wn*wn]); % first comma is numerator, demo is second

% generate frequency array of interest [rad/s]%
w = logspace(-1,2,1000)'; % logarithmic, from 1e-1 to 1e2 with 1000 points

% method 1: directly calculate frequency dependent complex gain %
% ===== %
jw = j*w; % j*w vector
r2d = 180/pi; % radians to degrees conversion factor
```

```

numerator = G.num{1}; % extract numerator coefficients [0 0 25]
denominator = G.den{1}; % extract denominator coefficients [1 0 25]
Gf = polyval(numerator,jw)./polyval(denominator,jw); % evaluate complex gain

% generate Bode plot %
figure(1); clf; zoom on;
loglog(w,abs(Gf),'b');
title('Bode Plot'); ylabel('Magnitude [ ]'); grid on;

figure(2);
semilogx(w,r2d*angle(Gf),'b');
ylabel('Phase [deg]'); xlabel('Frequency [rad/s]'); grid on;

% add collected data points on top of bode plot %
figure(2)
hold
plot(0.5,-18,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(1,-18,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(2.5,-22.5,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(5,-54,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(10,-90,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(25,-153,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(50,-162,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
figure(1)
hold
plot(0.5,0.75,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(1,0.85,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(2.5,1.2,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(5,1.15,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(10,0.2,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(25,0.02,'s','MarkerFaceColor','b','MarkerEdgeColor','k')
plot(50,0.025,'s','MarkerFaceColor','b','MarkerEdgeColor','k')

```