UNIVERSITY OF WATERLOO

Faculty of Engineering

SCALED SEGWAY STUB AXLE SAFETY VERIFICATION

Project 1

ME-321 - Kinematics and Dynamics of Machines

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November 20, 2015

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Abstract

The purpose of this report is to analyze and verify the safety of a hollow SAE 1010 cold-drawn steel stub axle on a scaled down version of HT i180 Segway Cart designed by Chris McClellan. To verify the safety of the axle, a safety factor is calculated under static and dynamic loads.

The lowest factor of safety for static loading is 34.9484 due to shear stress. Additionally, the lowest factor of safety for dynamic loading is 9.6894, calculated by using Tresca Criterion. Both safety factors are calculated based on properties listed in Tables 1 and 2. Thus, the axle design is limited by fatigue endured by dynamic loading.

1 Introduction

1.1 Background

Dean Kamen designed a one person dynamically self-balancing transportation vehicle, Segway Personal Transporter (Figure 1). Its meant to be yet another solution to green-energy transportation and revolutionize the way that short distance travel is conducted. The Segway PT costs \$0.50 of electricity for a full day of transportation, which is much more efficient than a traditional vehicle. The motion of the Segway is controlled by the rider shifting their weight forward to backward, and using the handle bar to turn.

Figure 1: Model HT i180 Segway

A Mechatronics Engineer, Chris McClellan, at the University of Waterloo decided to scale down the HT i180 model to two thirds of the original to create an apparatus for demonstrations or labs as application of control theory. The scaled Segway is to hold a maximum payload of 35.66kg, compared to full size that holds 117kg. The drive system in the scaled model consists of Brushless DC motors (HBL-12” R-36), and wheels with internal stub axle.

1.2 Objective and Method

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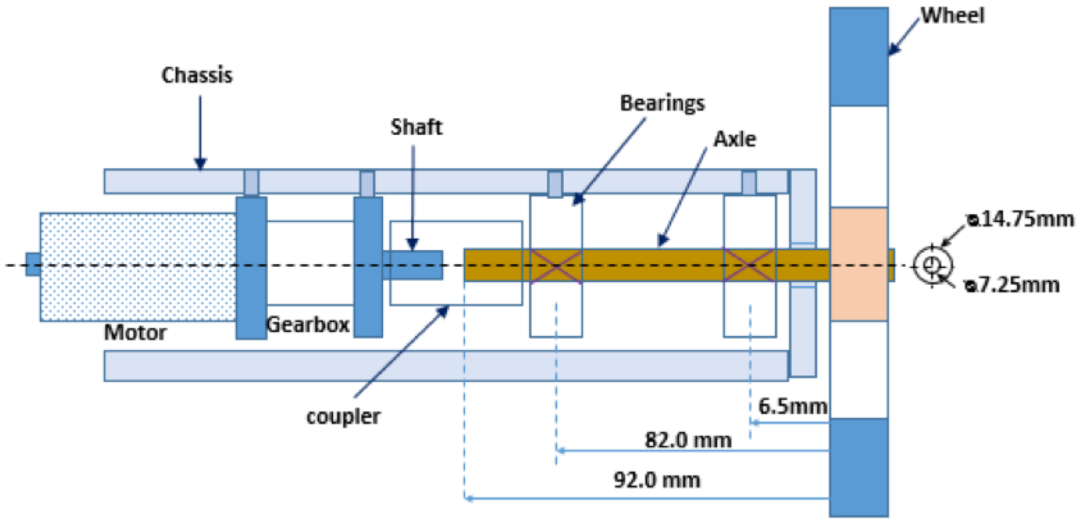

Figure 2: Drive Unit Schematic illustrating the stub axle

2 Summary of Results

2.1 Stub Axle Properties and Assumptions

Table 1 displays the properties of the stub axle in the scaled model of the Segway that will be used in calculations of the safety factors.

Table 1: Scaled Segway Stub Axel Properties

Outer Diameter	14.75mm
Inner Diameter	7.25mm
Material	SAE 1010 cold-drawn steel
Yield Strength σ_y	305 MPa
Ultimate Strength σ_u	365 MPa

The calculations are performed under multiple underlying assumptions:

1. Assume rigid body dynamics for static loading
2. Assume infinite service life of components for dynamic loads.
3. The effects of seal grooves and bearing seats on the axle are ignored
4. The force applied by the wheel and bearings on the axle are point forces, instead of distributed
5. Stresses due to torque induced by the motor are ignored
6. The force of the coupler is ignored
7. The mass of the platform and the max payload are evenly distributed between the two axles
8. The stub axle is wrought

2.2 Static Loading Analysis

Figure 3 illustrates the force loading and the dimensions of the axle. A and B are the bearing that the axle goes through and C is the wheel with the internal stub axle under analysis.

First, calculate the force applied on the wheel using half of the combined mass of the platform and the maximum payload of 35.66 kg.

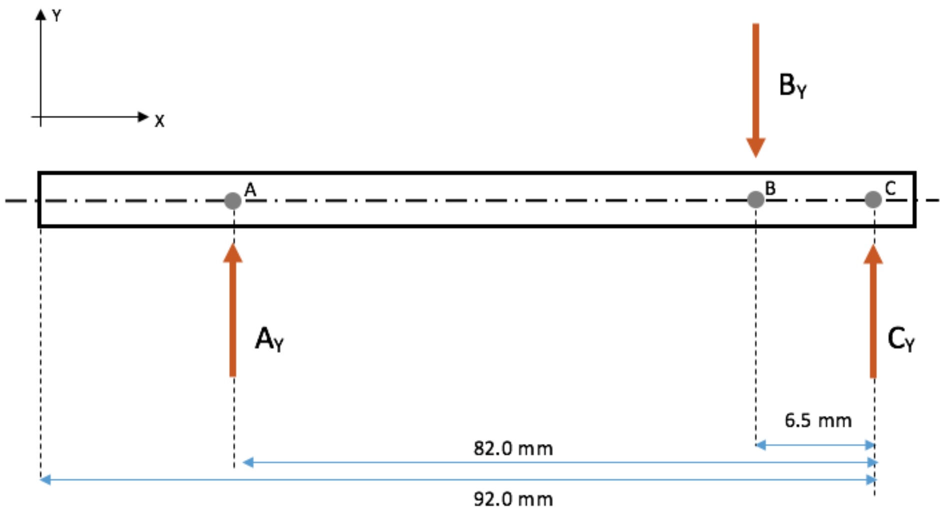

Figure 3: Loading diagram of the stub axle

$$
\begin{aligned}
C_Y &= \frac{(m_{platform} + m_{MaxPayload}) * g}{2} \\
&= \frac{(30.9 \text{ kg} + 35.66 \text{ kg})\,(9.81 \text{ m}/s^2)}{2} \\
&= 326.4768 \text{ N}
\end{aligned} \tag{1}
$$

Next, to find the force that bearing A exerts on the axle, about bearing B the moments are summed.

$$
\begin{aligned}
\sum M_{@B} &= 0 \\
0 &= A_Y \Delta X_{AB} - B_Y \Delta X_{BC} \\
A_Y &= B_Y \frac{\Delta X_{BC}}{\Delta X_{AB}} \\
A_Y &= 326.4768 \text{ N} \frac{6.5 \text{ mm}}{75.5 \text{ mm}} \\
A_Y &= 28.1073 \text{ N}
\end{aligned} \tag{2}
$$

Lastly, by the force equilibrium in the Y direction find B_Y.

$$
\begin{aligned}
\sum F_Y &= 0 \\
0 &= A_Y - B_Y + C_Y \\
B_Y &= A_Y + C_Y \\
B_Y &= 354.5841 \text{ N}
\end{aligned} \tag{3}
$$

Using the calculated forces the shear (Figure 4) and bending moment (Figure 5) diagrams are created.

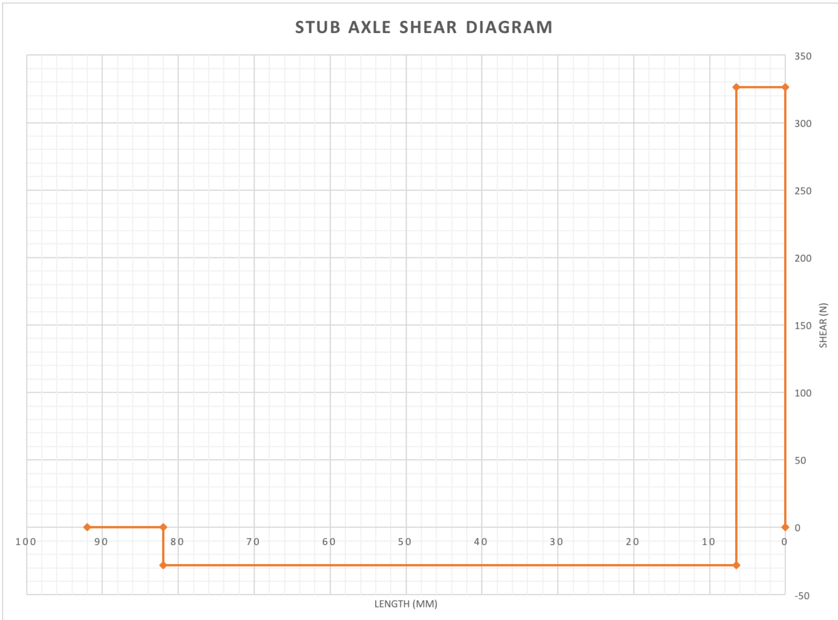

Figure 4: Shear force diagram over axle's length

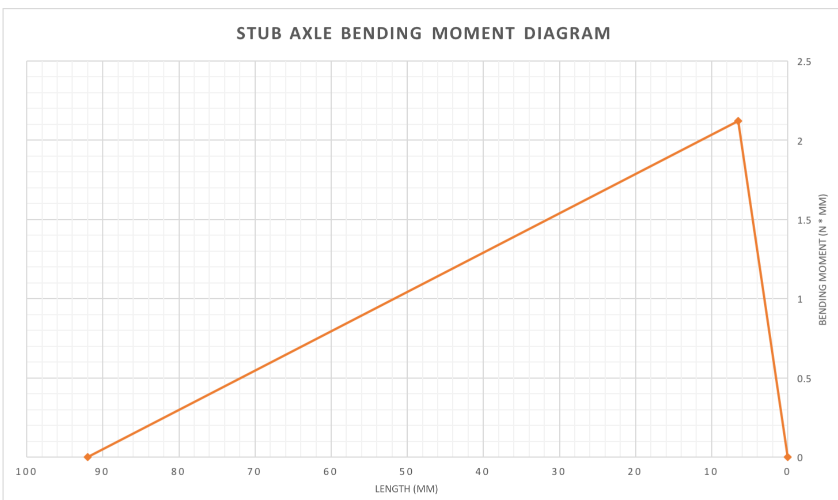

Figure 5: Bending moment diagram over axle's length

As seen from the graphs above the maximum shear force of $V = 326.4768\ N$ occurs from bearing B to the wheel (C), which is used to calculate maximum shear force that occurs at the neutral axis (Figure 6).

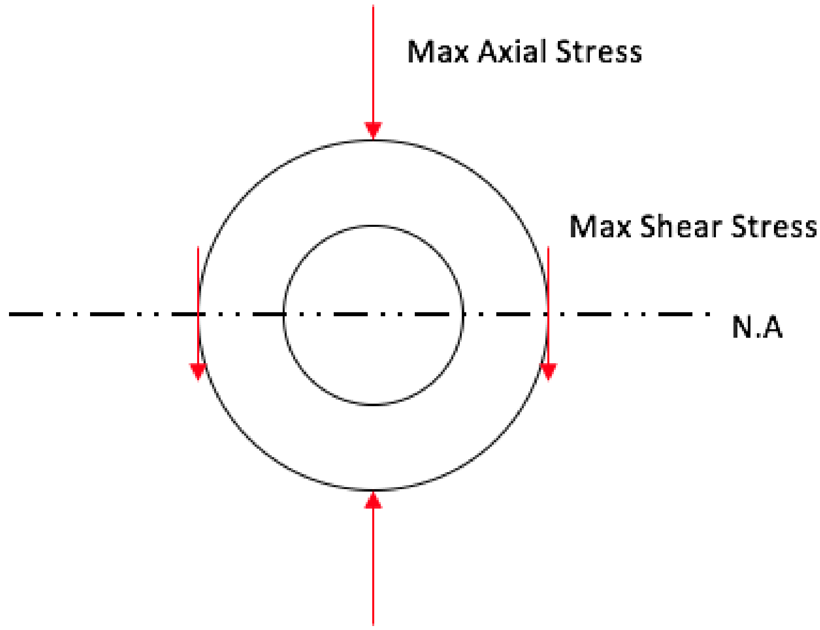

Figure 6: Stress location on the cross section at B

$$\begin{aligned}
\tau_B &= \frac{VQ}{It} \\
&\approx \frac{2V}{A} \text{ , A is the cross sectional area of the beam} \\
&\approx \frac{2V}{\pi\left(r_{out}^2 - r_{in}^2\right)} \\
&\approx \frac{2\,(326.4768 \text{ N})}{\pi\left(\frac{14.75 \text{ mm}}{2}^2 - \frac{7.25 \text{ mm}}{2}^2\right)} \\
&\approx 5.0386 \text{ MPa}
\end{aligned} \tag{4}$$

Additionally, the maximum bending moment of $M = 2.1221\ Nm$ occurs at point B, which is used to calculate the maximum stress due to bending that occurs at top and bottom of the axle (max distance away from neutral axis y) as illustrated in Figure 6.

$$\begin{aligned}
\sigma_B &= \frac{My}{I} \\
&= \frac{My}{\frac{\pi}{4}\left(r_{out}^2 - r_{in}^2\right)} \\
&= \frac{(2.1221 \text{ Nm}) * (1000)(\frac{14.75 \text{ mm}}{2})}{\frac{\pi}{4}\left(\frac{14.75 \text{ mm}}{2000}^2 - \frac{7.25 \text{ mm}}{2000}^2\right)} \\
&= 7.1533 \text{ MPa}
\end{aligned} \tag{5}$$

The two stress elements are considered to determine principle stresses using Von Mises method. As mentioned above, the maximum axial stress occurs at maximum distance from the neutral axis at the top and bottom of the axle. Therefore, in this scenario there is only axial stress in the x direction with no shear force present. Thus, the maximum principle stress is equal to stress due to bending, σ_B.

$$\begin{aligned}
\sigma_{(max,min)} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma y}{2}\right)^2 + \tau_{xy}^2} \\
&= \frac{\sigma_x + 0}{2} \pm \sqrt{\left(\frac{\sigma_x + 0}{2}\right)^2 + 0} \\
&= \sigma_B \text{ , shown in Equation 5}
\end{aligned} \tag{6}$$

Using the Von Mises yield criterion to calculate the safety factor assuming rigid body dynamics.

$$\begin{aligned}
SF &= \frac{\sigma_y}{\sigma_{max}} \\
&= \frac{\sigma_y}{\sigma_B} \\
&= \frac{305 \text{ MPa}}{7.1533 \text{ MPa}} \\
&= 42.6374
\end{aligned} \tag{7}$$

The second stress element lies on the neutral axis, at which point the shear is maximum and no axial stress is present.

$$
\begin{aligned}
\sigma_{(max,min)} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma y}{2}\right)^2 + \tau_{xy}^2} \\
&= \frac{0+0}{2} \pm \sqrt{\left(\frac{0+0}{2}\right)^2 + \tau_B^2} \\
&= \tau_B \text{ , shown in Equation 4}
\end{aligned} \tag{8}
$$

Again using the Von Mises yeild criterion which states that the magnitude of the shear stress in pure shear is $\sqrt{3}$ times lower than the tensile stress in the case of simple tension.

$$
\begin{aligned}
SF &= \frac{\sigma_y}{\sigma_{max}} \\
&= \frac{\sigma_y}{\sqrt{3}\tau_B} \\
&= \frac{305 \text{ MPa}}{\sqrt{3} * 5.0368 \text{ MPa}} \\
&= 34.9486\text{MPa}
\end{aligned} \tag{9}
$$

Therefore shear is the dominant method of static loading for the stub axle.

2.3 Dynamic Loading Analysis

The fatigue of the axle is due to the rotating shaft which is endures repeated and reversed stress. This creates a cyclic load of zero mean, and maximum amplitude of $\sigma_a = 7.1533$ MPa and $\tau_a = 5.0386$. The endurance limit for wrought cold-drawn steel with $\sigma_u = 365$ MPa is approximately $s_n = 140$ MPa from Figure 5-8 on page 172 [1]. To calculate the actual endurance limit Equation 10 is used.

$$
s_n' = s_n C_m C_{st} C_R C_s \tag{10}
$$

Table 2 summarizes the axle material properties for calculating the endurance limit.

Table 2: Axle Endurance Material Properties

C_m	1.0	flaws in manufacturing in wrought steel
C_{st}	1.0	bending stress
C_R	0.75	reliability factor for 99.9% reliability Table 5-2 [1]
C_s	7.62 < D ≤ 50	D = 14/75, size factor, Table 5-3 [1] Equation 11

$$\begin{aligned} C_s &= (\frac{D}{7.62 \text{ mm}})^-0.11 \\ &= (\frac{14.75}{7.62 \text{ mm}})^-0.11 \\ &= 0.9299 \end{aligned} \tag{11}$$

Thus using Equation 10 above the actual endurance limit is calculated below (Equation 12.

$$\begin{aligned} s'_n &= (140 \text{ MPa})\,(1.0)\,(1.0)\,(0.75)\,(0.9299) \\ &= 97.6421 \text{ MPa} \end{aligned} \tag{12}$$

The principle stress σ'_a is equal to σ_a due to only axial stress at max distance away from the neutral axis, which causes the bending of the axle. Von Mises Criterion is then used to calculate the safety factor for zero mean dynamic load.

$$\begin{aligned} K_t\sigma'_a &= \frac{s'_n}{N} \text{ , assuming no stress concentrations } K_t = 1 \\ &= \frac{97.6421 \text{ MPa}}{7.1533 \text{ MPa}} \\ &= 13.6499 \end{aligned} \tag{13}$$

The principle stress τ'_a is equal to τ_a as shown before. Tresca Criterion is then used to calculate the safety factor for zero mean dynamic load.

$$\begin{aligned} K_t\tau'_a &= \frac{s'_n}{2N} \text{ , assuming no stress concentrations } K_t = 1 \\ &= \frac{97.6421 \text{ MPa}}{2 * 5.0386 \text{ MPa}} \\ &= 9.6894 \end{aligned} \tag{14}$$

3 Discussion

In reality the safety factor for both dynamic and static loads will be lower as the assumptions ignore torque and assume perfect manufacturing process which omits all stress concentrations of seal grooves and bearing seats. However, considering all the assumptions, the axle can be evaluated as over designed and will not be failing component of the Scaled Segway design.

Furthermore, the verification determined that the stub axle will fail under dynamic loading conditions rather than static, as shown by the 3/4 decrease of the safety factor from static loading analysis.

Lastly, it was assumed that the principle stresses will occur at either the neutral axis or at the top and bottom (max distance away from the neutral axis). However, it is possible that there exists a combination of the two stresses that creates a greater maximum principle stress resulting in lower safety factor.

4 Conclusion

The factor of safety for static loading of the scaled Segway SAE 1010 cold-drawn steel stub axle is 34.9484 and fails due to shear stress. The factor of safety for dynamic loading is 9.6894, calculated by using Tresca Criterion. Therefore the axle design is limited by fatigue endured by dynamic loading. The safety factors reflect correct results, based on the assumptions made before the calculations were done. Assuming all the assumptions are realistic and correct the conclusion to the safety verification is that the axle is over designed and can be reduced in weight to have a lower safety factor to save cost.

5 References

[1] P. Robert L. Mott, *Machine Elements in Mechanical Design.* Pearson, 2014.