UNIVERSITY OF
WATERLOO

ECE 484

Lab 2 Report

## Design of the Inner Loop Controller and Modeling of the Ball and Beam

Group 28 - Station #8

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1.0 Introduction

This lab is part of a project that focuses on creating a control system to position a ball on a beam by controlling the beam incline. The beam consists of two parallel rods that form a potentiometer with the ball to determine the ball's position. The beam is controlled by a DC motor through a gear system and a lever arm (see Figure 1 below).
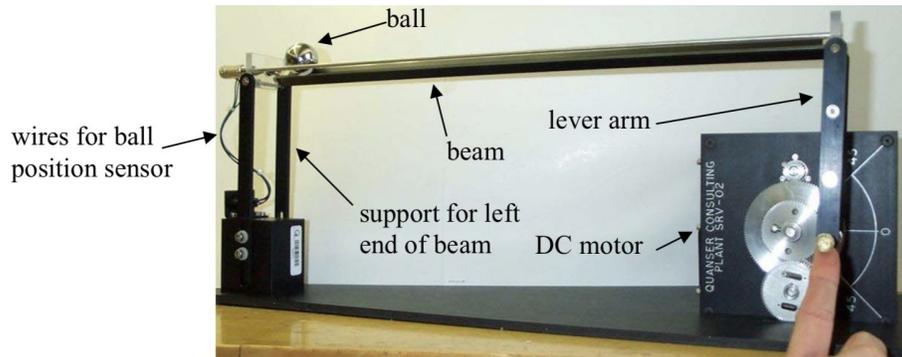
*Figure 3: The plant: a "ball and beam" apparatus. [1]*

Essentially the ball will be controlled by voltage of the motor that is converted into an angle by the control system that is created in LabView and GWiz plug in. The mathematical theory can be observed in the diagram below (Figure 2).
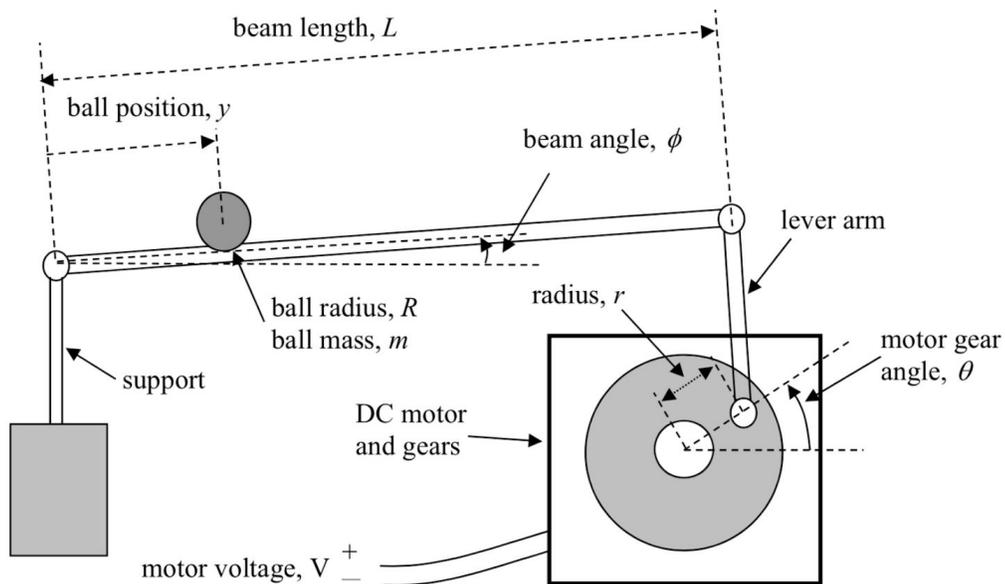
Figure 4: The plant: a "ball and beam" apparatus. [1]

This lab focuses on designing the inner loop controller and verifying the parameters of the plant model (see Figure 3). In particular, the emulation design process is used to design a discrete-time

controller for driving the position of the motor gear angle. To verify the plant model, an experiment is performed to ensure that the model for the ball on the beam is accurately represented by the plant model parameters shown in Figure 3.
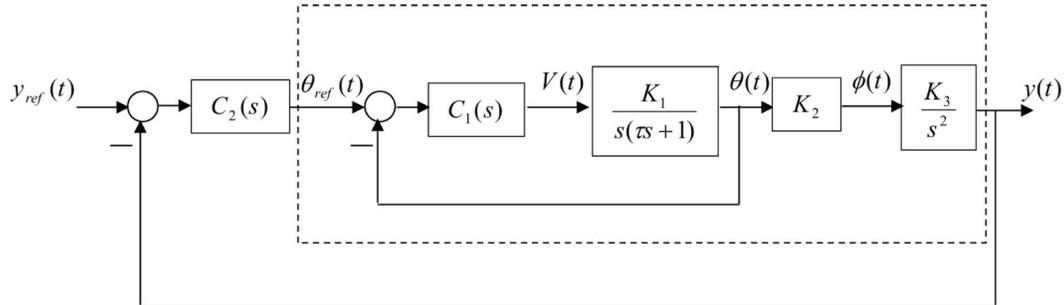

Figure 3. Inner/outer loop control configuration for the linearized plant.[1]

2.0 Inner Loop Controller Design

The emulation design approach involves first designing a continuous time controller for our system using the continuous time plant model already developed for the motor. To do this, a lead compensator is designed for the system that meets the following specifications outlined in the lab manual:

- The step response steady-state tracking error is zero
- The step response 2% settling time is no more than 0.50 seconds
- The step response overshoot is no more than 5%
- The motor voltage does not saturate.

The design of the lead compensator was done using Simulink. A model of the inner loop system with a lead compensator was created, as shown in Figure 4, and values were selected for a and b until the response of the system fell within the required specifications.
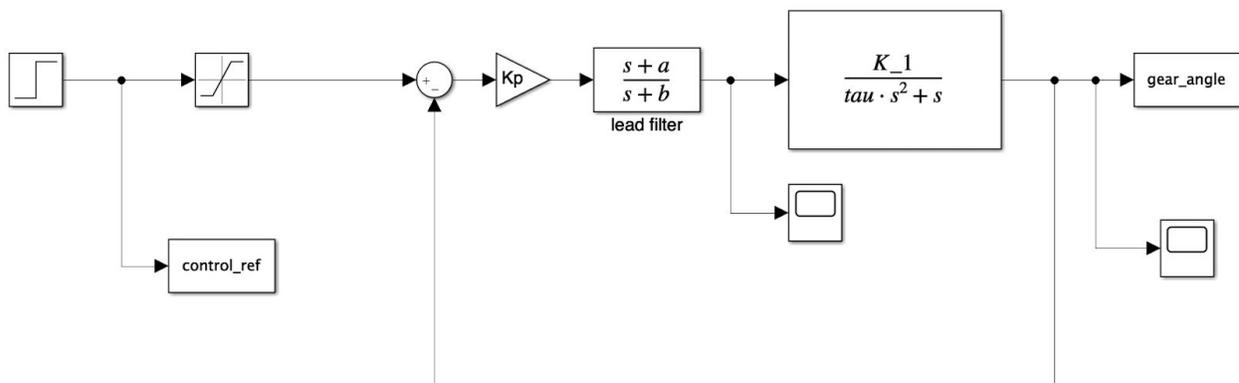
Figure 4. Simulink model used to develop lead compensator

The final values of the controller gain, a and b that gave the desired response are -8, 20 and 25, respectively. Figure 5 and 6 below shows the step response of the motor gear angle and the motor voltage control signal for a 0.7 radian control input. As can be seen, all specifications outlined earlier are met.
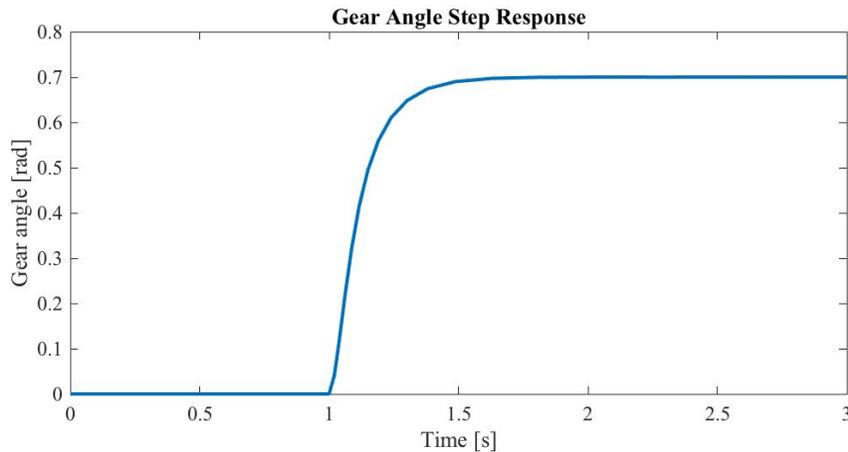

Figure 7: Step response of the gear angle using the designed lead compensator
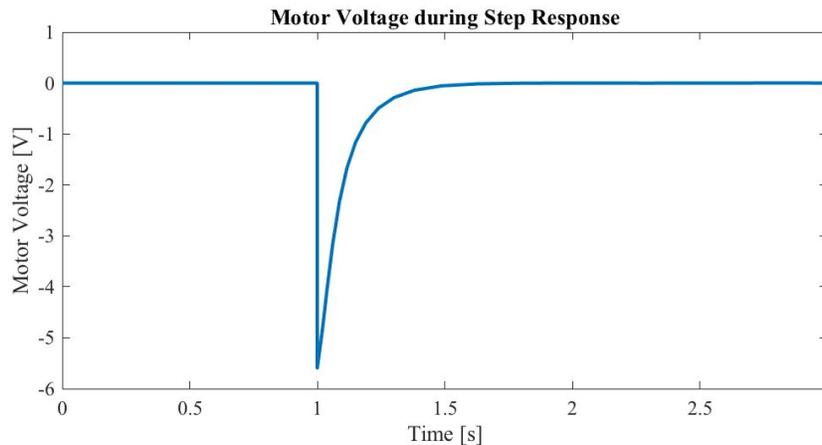
Figure 8: Motor voltage applied during the step response simulation

3.0 Controller Discretization

The lead compensator controller is then discretized using the emulation approach. To do this, MATLAB was used, the code for which is included in the Appendix. To arrive at a sampling period, the bandwidth of the above controller was determined using the âá^ä}↔ä\åÁ function in MATLAB. This bandwidth was then multiplied by 25 and then inverted to obtain the desired sampling period of 0.02726 seconds. The final discrete-time controller outputted by the MATLAB code is as follows:

$$E_f = \frac{-7.593z + 4.34}{z - 0.4917}$$

The coefficients YáÊÁâÊÁ´ÊÁäŸ to full 15-digit precision are included in the Formula Node Code in the Appendix.

This discrete time controller was also implemented in Simulink, as shown in Figure 7 below.

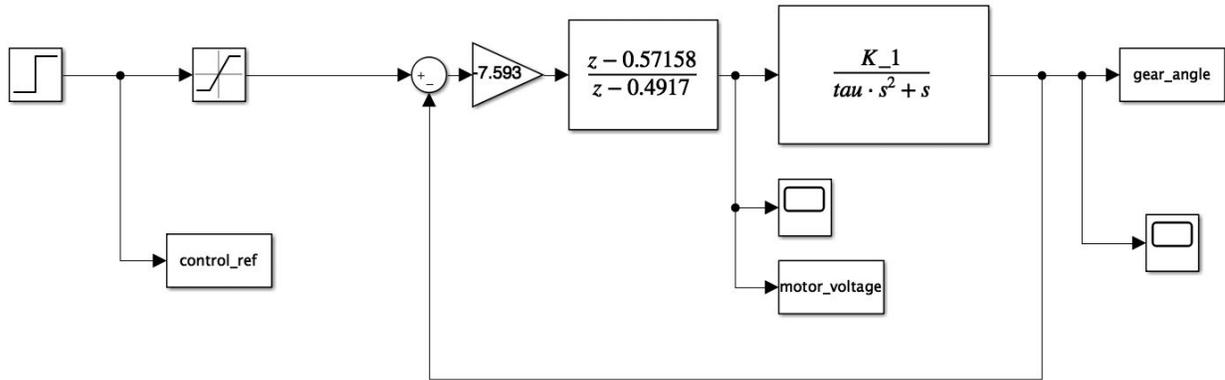*HKWTG'9‹*Simulink model including the discrete-time controller

The step response of this system was also obtained, as shown in Figures 8 and 9. It is evident that the performance of the discrete time controller is very close to that of the continuous time controller on which it is based. Thus meeting all the design requirements.
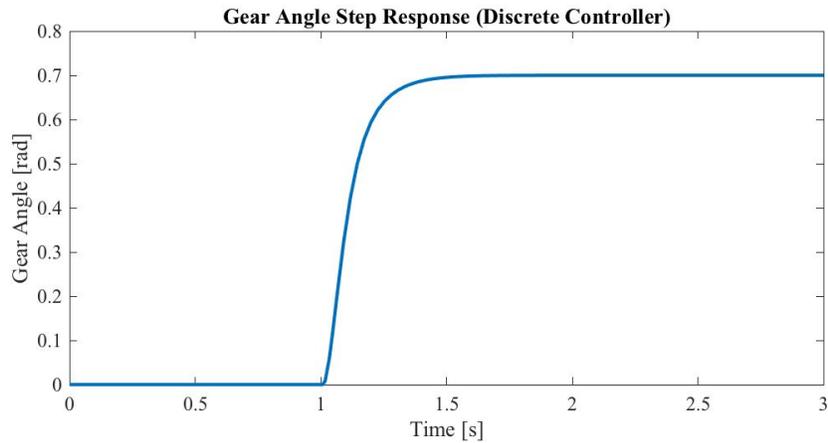
*HKWTG'‹*Step response of the discrete-time controller
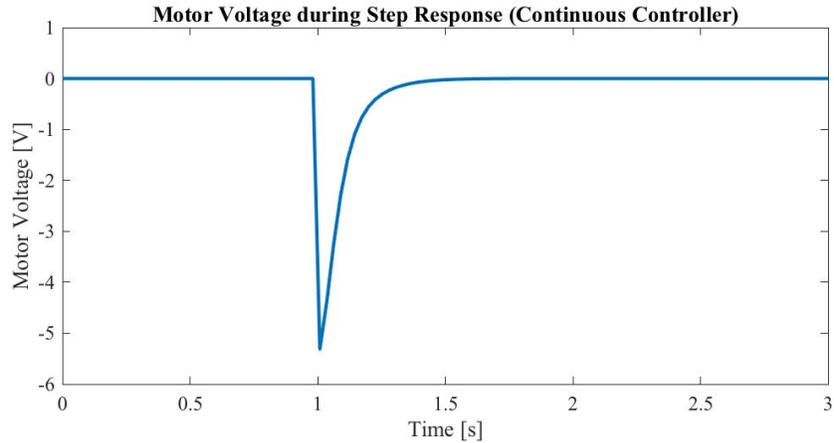
Figure 9: Motor voltage applied by the discrete-time controller

4.0 Discrete Controller Implementation

The step input response of the implemented discretized controller is shown in Figure 10. The controller implementation guide included in the Appendix of the lab manual was used to develop the controller. The code for the controller implementation is included in the Appendix of this report. Comparing to the experimental in Figure 5 the response is almost identical, thus meeting all the required specifications and is allowed to be used with the ball and beam modelling system described in the following section.
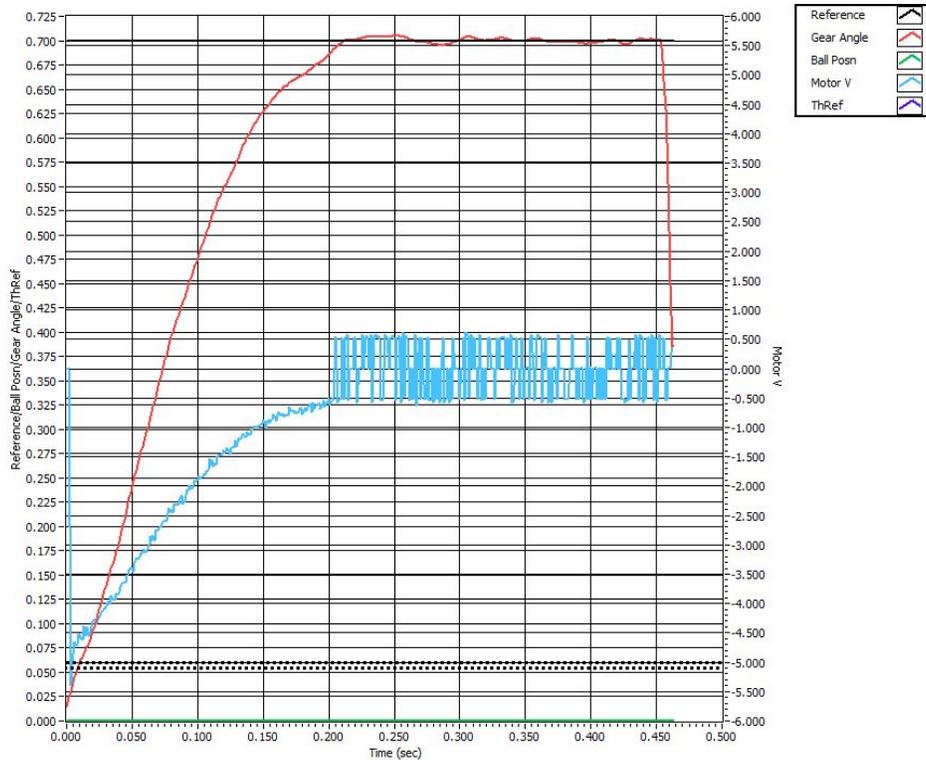
Figure 10: Test of a step input through the discretized controller

5.0 Ball and Beam Modelling

Attaching beam arm to the motor plant influences the inner-loop behaviour as it changes its characteristics and thus should be remodelled. However, due to high gearing ratio the model does not change significantly hence the model remains the same. The stiction offset value remained at 0.480 V for clockwise rotation but changed to -0.820 V for counter-clockwise rotation. Figure 11 demonstrates the inner loop response to a square wave which still meets the desired requirements in both directions. The only difference is that the motor voltage saturates in the counterclockwise direction, because the stiction offset is much higher than before. The saturation is very brief and small in magnitude, and it does not impact the step response significantly.
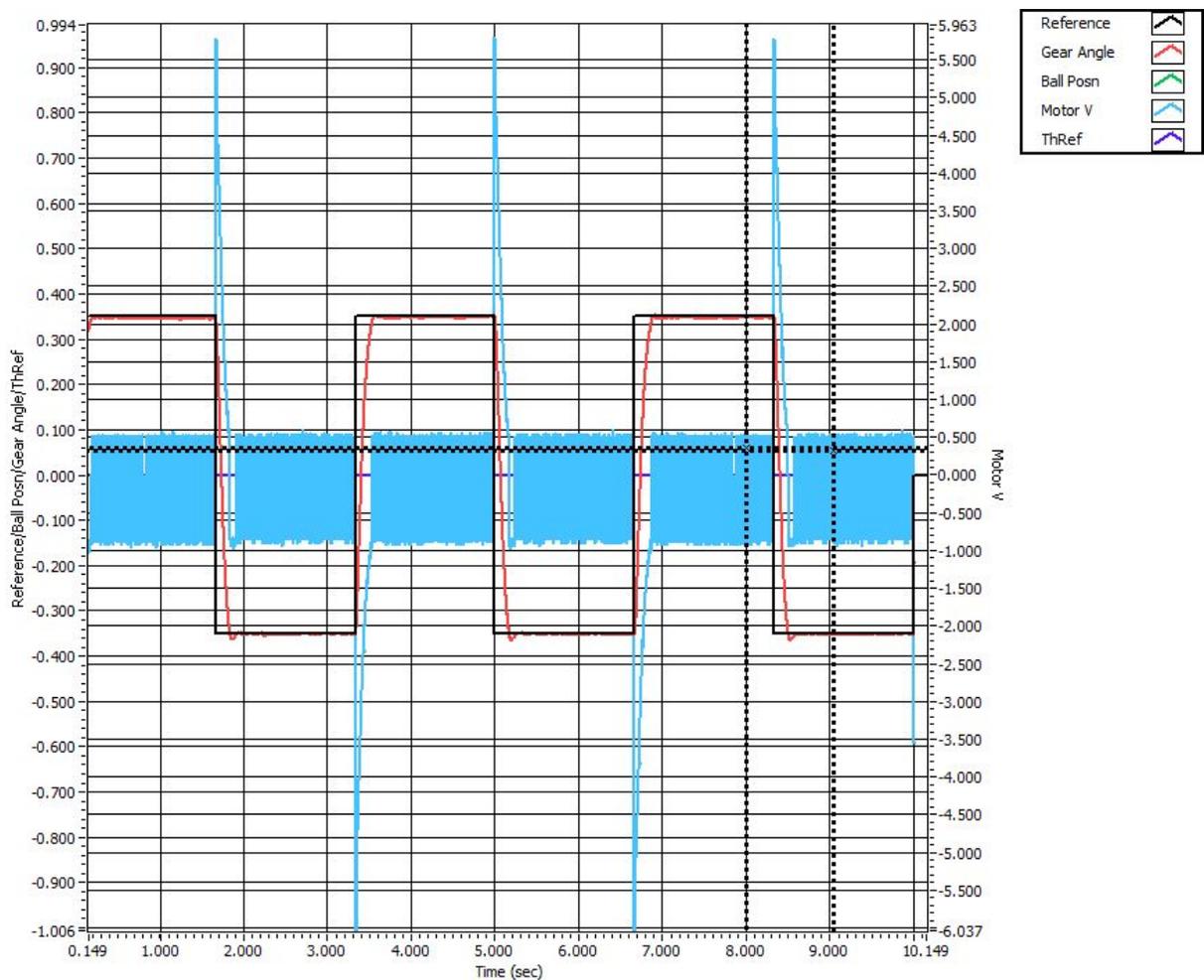
Figure 11: Multi-step response of the inner-loop controller with modified stiction values

6.0 Ball Position Scaling

The ball position scaling was determined by taking the potentiometer voltage readings at the ball's max and min positions on the beam, these are summarized in Table 1.

Ball Position [cm]	Potentiometer Voltage [V]
41.7	7.25
0	3.266

The scaling is then determined by a simpleÁ linear interpolation between the two points, done using excel. Figure 12 below shows the excel plot and the linear equation obtained using the data from Table 1. The final relationship is:

âá→→Ž*~bÁKÁF€ÈHıJÍÁEÁ*~\Ž{~→\á&æÁËÁĞHÈFÎIÁ

BALL POSITION SCALING

$y = 10.467x - 34.185$

Y-axis: BALL POSITION [CM] (0 to 45)
X-axis: POTENTIOMETER VOLTAGE [V] (0 to 8)
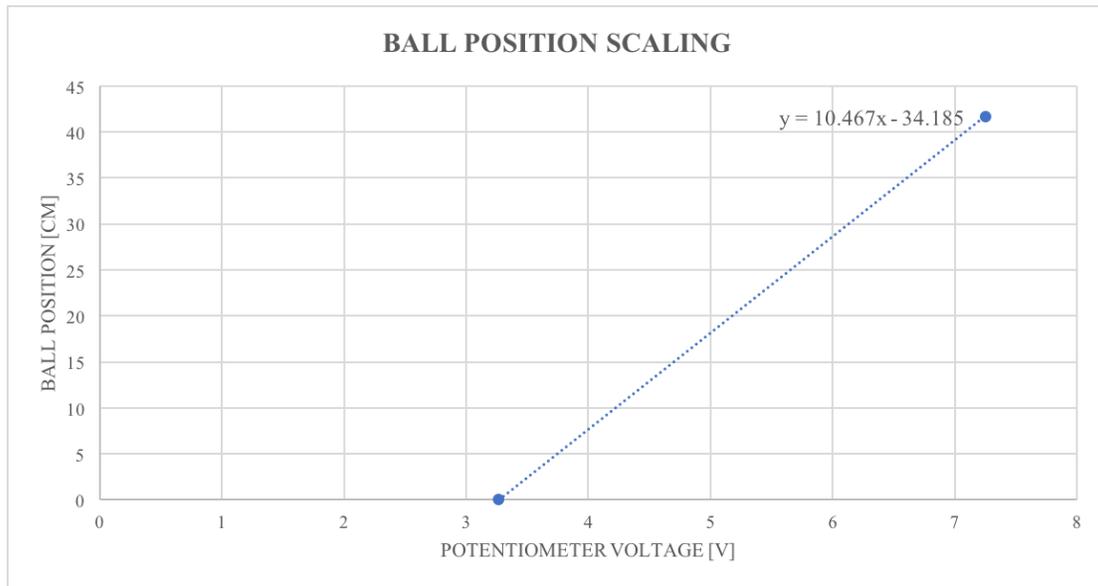
*HK̆ WTG'34∢*Motor voltage applied by the discrete-time controller

7.0 Outer Loop Block Diagram

Figure 13 below shows a fully detailed block diagram of the outer loop of the controller, including the inner loop diagram developed in Lab 1.
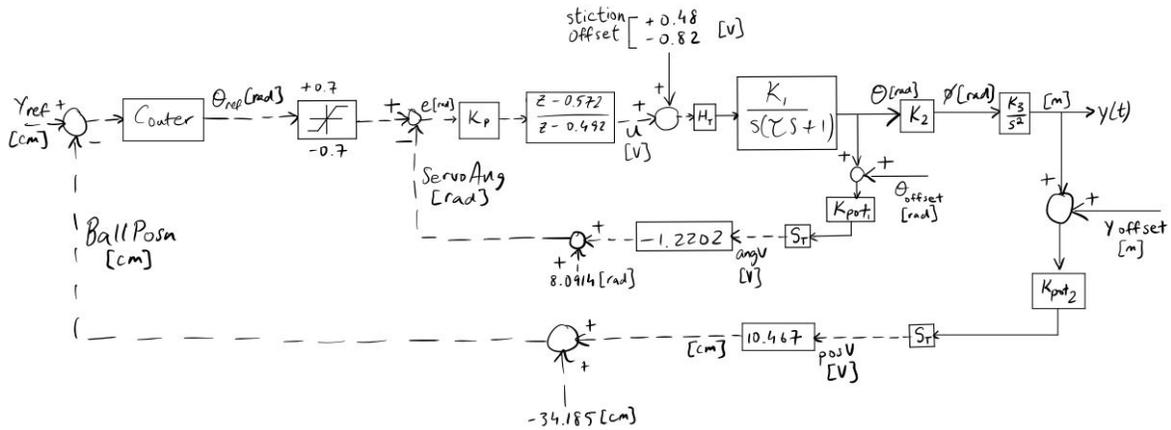
*HK WTG'35<*Block diagram of outer loop, including inner loop

8.0 Plant Model Parameter Verification

The motor angle transformation to the lever angle is modeled by the following equation:

$$\frac{\phi(u)}{\Theta(u)} = M_2, \ y \ j \ gt \ g \ M_2 = 0.062$$

This coefficient is determined through the geometry of the system, which is fixed, and thus does not require verification. The ball position in meters is determined from the lever angle by the following relationship:

$$\frac{l(u)}{\phi(u)} = \frac{M_3}{u^2}, \ \ y \ j \ gt \ g \ M_3 = 4.78 \ [o \ /u^t]$$

This relationship is dependant on many physical characteristics that determine how the ball rolls across the beam. There is potential for significant variation in these characteristics, so verification of this coefficient is required.

To verify K_3, an experiment is performed with the apparatus where the beam angle is held at a fixed position of 0.562 radians and the ball is allowed to roll from one end of the beam to the other. Figure 14 below shows the results of this experiment, with the ball position shown in green vs time.
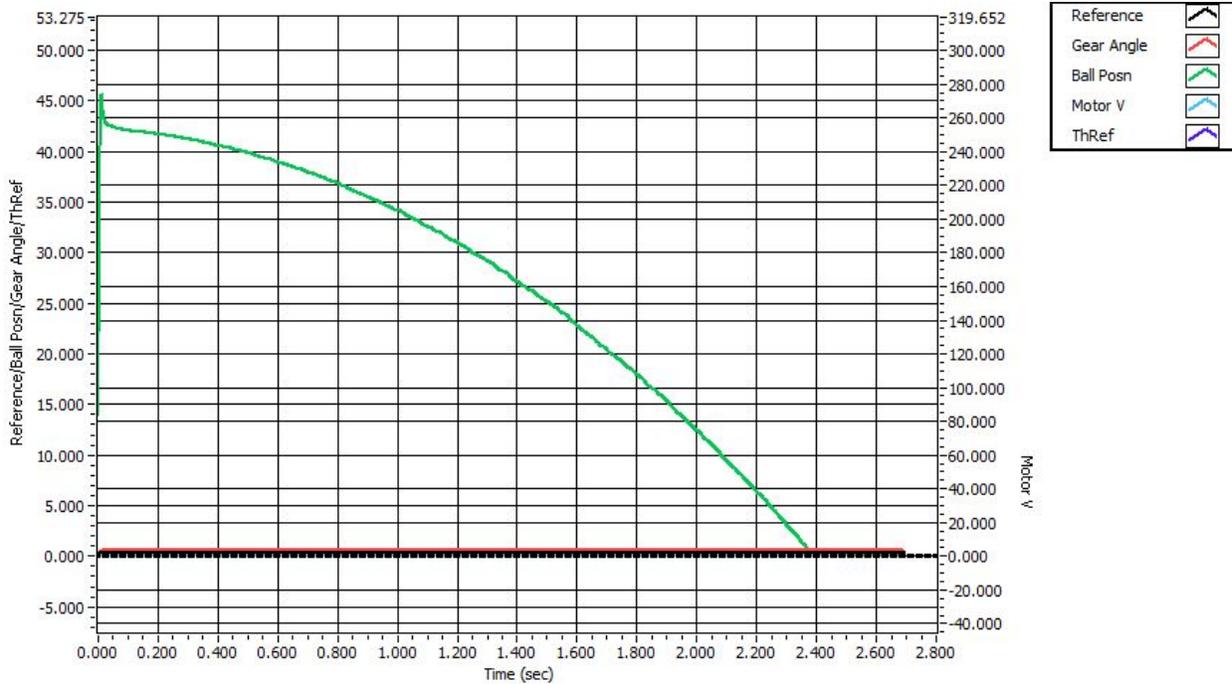
Figure 36 – Ball position vs. time at a fixed angle of 0.562 rad

To verify this, the inner loop controller and open loop plant configuration is recreated in simulation using the given K_2 and K_3 values. The simulink model is shown below in Figure 15.
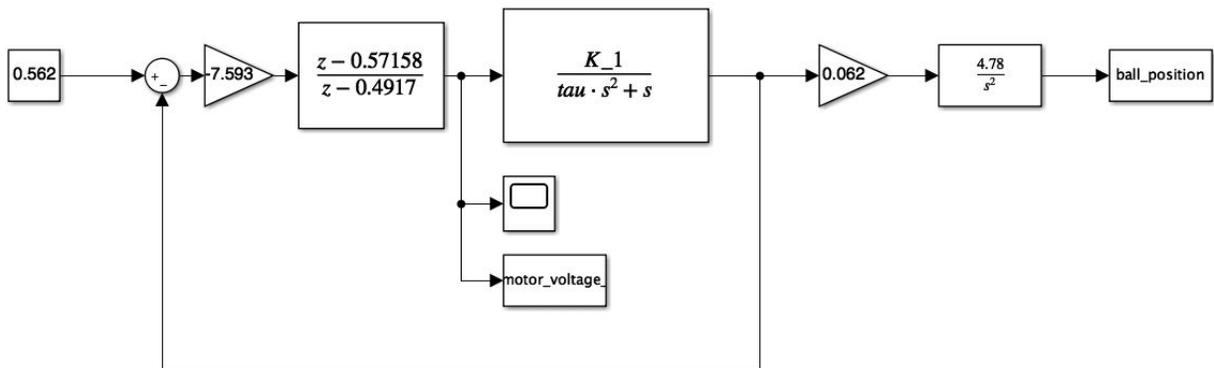
Figure 37 – Simulink model of inner loop controller with open loop plant

The output of this simulation, using the same input angle of 0.562 radians as the experiment, is shown below in Figure 16. The simulation result is scaled to convert the meters output of the plant to centimeters. Additionally, an offset is added that is equal to the length of the beam (41.7cm) and a negative one multiplier is applied to replicate the fact that the ball rolls on the beam from its maximum y position to zero.

9
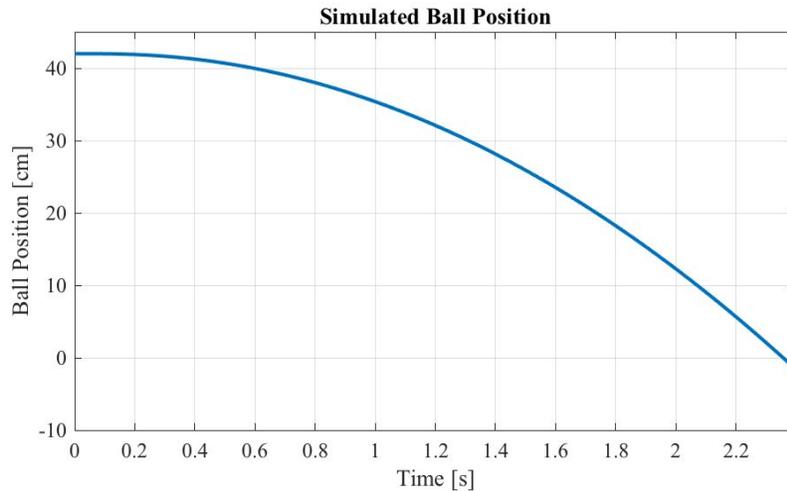
*H/̄ WTG'38<*Simulated ball position at a fixed angle of 0.562 rad

Comparing the experimental results and the simulation results, it is clear that they match very closely. Both curves are the exact same shape and take the same amount of total time (2.35 seconds) to travel across the 41.7 cm of the beam length. This suggest that the K_3 value given is accurate and does not require any modification in order to model our plant.

9.0 Conclusion

The inner loop continuous time controller is developed according to time domain specifications listed:

- The step response steady-state tracking error is zero
- The step response 2% settling time is no more than 0.50 seconds
- The step response overshoot is no more than 5%
- The motor voltage does not saturate.

The controller is then discretized, verified that the performance did not change significantly, and successfully implemented to control the angle of the lever arm. Furthermore, the stiction offset values are adjusted to account for the lever arm, the beam and the ball entering the system. Next, the scaling slope for the ball position is determined to be âá→→Ž*~bÁ KÁ F€ÈHIJÍÁ EÁ *~\Ž{~→\á&æÁ ËÁ ĞHÈFÎI. Lastly, the parameter K_3 of the plant model is verified by comparing values of ball position obtained through experimentation and simulation.

The final system controller values are summarized in Table 2. The only values that were adjusted are the stiction friction to account for the beam, lever and ball attached to the motor.

*VCDNG'4<*Controller Values

Parameters	Old Values	New Values
K_1	-1.23	-1.23
τ	0.026 s	0.026 s
$CW_{stiction\_friction}$	0.480 V	0.480 V
$CCW_{stiction\_friction}$	- 0.480 V	-0.820 V
K_2	n/a	0.062
K_3	n/a	4.78 m/s^2

10.0 Acknowledgements

"The authors of this report declare that, in doing the lab work and writing up the lab report for ECE484, we followed rules 2, 3, 4, and 5 described at the beginning of the lab manual."

Rishab Sareen   Pavel Shering

Works Cited

[1] *Dcm ́cpf ́Dgco ́Rtqlgev: ́Uwf gpv ́J cpf qw.* Report. ECE, University of Waterloo. 2017.

Appendix

Formula Node Code:

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ÐEÁKKKKKKKKKÁÛUÓÞÁØSÚÓÞ ́ÔNOÓÁÚÓRŞQNÚÓÁKKKKKKKKKKKKKKÁEÐÁ
ÐEÁÁØ^bæã\Áâæ→~}Á\åæÁ ́~äæÁà~ãÁ]~|ãÁb ́á→↔^&ÊÁbá\|ãá\↔~^Áâ→~ ́←ÊÁá^äÁ
 ́~^\ã~→→æãbÈEÐÁ
Á
ÐEÁÁÜáã↔áâ→æbÁ↑á]ÁâæÁäæ ́→áãæäÁ~^Á\åæÁâ~[Áâ~ãäæãÊÁábÁbå~}^Áà~ãÁ\åæÁ↔^*|\ÁÁ
ÁÄÚ↑bÄÁá^äÁ\åæÁ~|\*|\ÁÄÑá→→Ş~b^ÄÈÁÜáã↔áâ→æbÁ ́á^Áá→b~Áâæ Áäæ ́→áãæäÁ↔^→≫^æÁábÁ}ábÁ
ä~^æÁà~ãÁÄÚæ↑*FÄÈÁEÐÁ
à→~á\ÁÚæ↑*FÌÁÁ
à→~á\ÁæÖæáãN^&ÌÁ
Á
ÐÐ ́~^\ã~→→æãÁ ́~^b\á^\bÁÁ
à→~á\ÁáÁÁKÁÁËÍÈIÏĞĞIÍFĞF€F€ÎĞÍÌÁ
à→~á\ÁáâÁÁKÁÁHÈĞH€GFHFÍÏ€ÏÍIĞIÌÁ
à→~á\Á ́ÁKÁÁFÌÁ
à→~á\ÁäÁÁKÁË€ÈHÏFIJÏIJHFĞÍIJĞIHIJÌÁ
Á
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ÐEÁÁUå↔à\Áãæ&↔b\æãbÁ*æã↑↔\Á*ãæ{↔~|bÁ{á→|æbÁ~àÁ{áã↔áâ→æbÁ\~ÁáæÁbá{æÈÁÁ
ÚåæÁ~|*|\Á{áã↔áâ→æÁÄÁæÄÁ↔bÁ}↔ãäÁ\~ÁáÁbå↔à\Áãæ&↔b\æãÁ↔^*|\Á~^Á\åæÁÔ~ãÁQ~~*Á
â~ãäæãÈÁ Á

ÚåæÁ↔^*|\bÁÄæFÁÁá^äÁÄæGÁáãæÁ}↔ãæäÁÁ\~Á\åæÁ´~ã㜇*~^ä↔^&Ábå↔à\Áãæ&↔b\æãÁ~|*|\bÈÁ
ÁÄæFÁÁå→äbÁ\åæÁ{á→|æÁ~àÁÄæÄÁãã~↑Á\åæÁ*ãæ{↔~|bÁ↔\æãÁ\↔~^Áá^äÁÄæGÁÄå→äbÁ\åæÁ
{á→|æÁ~àÁÄæFÁÁã~↑Á\åæÁ*ãæ{↔~|bÁ↔\æãÁ\↔~^ÈÁEÐÁ

Á

ÐEÁŞ→á´æÁ]~|ãÁbæ^b~ãÁUONQØSÖÁåæãæÁEÐÁ
ÐEÁSŠÁb´á→↔^&Á↔bÁ*ã~{↔äæÄÁà~ãÁ\åæÁäæ↑~ÁEÐÁ
Ñá→→Ş~b^ÁKÁF€ÈHıJÍE*~bÜÁÉÁĞHÈFÎıÌÁÁÐEÁÜÁ\~ÁÜÁEÐÁ
Uæã{~N^&ÁKÁÇÉFÈGG€GEá^&ÜDÁÉÁÎÈ€ÏFHÌÁÐEÁÜÁ\~ÁãáäÁEÐÁ
ÐEÁUONQØSÖÁæ^äÁEÐÁ

Á

↔àÁÇQ~~*ÁÁJÁĞDÁÐEÁá→→Ábå↔à\Áãæ&↔b\æãbÁ´→æáãæäÁÁà\æãÁĞãäÁ↔\æãá\↔~^ÌÁ\å↔bÁ
b\á\æ↑æ^\Á↔^^↔\↔á→↔ˇæbÁ\åæÁbå↔à\Áãæ&↔b\æãbÁEÐÁ
ÁÁ¦|ÁKÁæÁKÁÚåÞæàÁKÁ*~bÜKÁá^&ÜÁKUæã{~N^&ÁÑá→→Ş~b^KÁ€ÌcÁ
æ→bæÁ
¦Á
ÁÁÁ↔àÁÇRá^|á→DÁÁÁÐE↑á|á→Á↑~\~ãÁ{~→\á&æÁ´~^\ã~EÐÁ
ÁÁ|ÁÁ|ÁKÁR~\ÜÌcÁ
ÁÁÁæ→bæÁÁÁÁÐE´~^\ã→Áá→&~ã↔\å↑EÐÁ
ÁÁ|ÁÁ
Á
ÁÁÁÐEÁONÛÚøŠSıÁŒŠÁSŠÚÁ→~áäÁ\åæÁ~|*|\Á~àÁÁ^~^↔↔^æáãÁâ→~´←ÁÇæÈ&ÈÊÁbá\|ãá\~ãÊÁ
~àâbæ\DÁ↔^\~ÁáÁUÒ ØÔÚÁÞÓÖØUÓÔÊÁÁ
ÁÁÁ\~Áá{~↔äÁ↔^\ã~ä|´↔^&ÁÁ^~^↔↔^æáã~\]Á↔^\~Á]~|ãÁ´~^\ã~→æãÁ→~~*ÈÁOãæá\æÁ
bæ*áãá\æÁ{áã↔áâ→æbÁ\~Áå→~äÁ^~^↔↔^æáã{á→| æbÈEÐÁ
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ÁÁÁÐEÁŞ→á´æÁ]~|ãÁ~|\æãÁ→~~*ÁÑNQQÁŞŠUØÚØŠÁOŠSÚÞŠQQÓÞÁâæ→~}ÁEÐÁ
ÁÁÁÑá→→Ş~b^ÁKÁ€ÌÁÐÐÁÞÓRŠÜÓÁ\å↔bÁ→↔^^æ}áæ^Á\åæÁâá→Á~↔bÁæ~^&Á|bæäÁ~^Á\åæÁâæá↑Á
Á
ÁÁÁÐEÁŞ→á´æÁ]~|ãÁ&æáãÁá^&→æÁUNÚÛÞNÚŠÞÁâæ→~}ÁEÐÁ
ÁÁÁ↔ÇãæàÁJÁ€ÈÍDÁ¦Á
ÁÁÁÁÁ Á ãæàÁKÁ€ÈÍÌÁ
ÁÁÁcÁ
ÁÁÁæ→bæÁ↔àÁÇãæàÁLÁ€ÈÍDÁ¦Á
ÁÁÁÁÁÁÁ Á ãæàÁKÁ€ÈÍÌÁ
ÁÁÁcÁ Á
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ÁÁÐEÁŞ→á´æÁ]~|ãÁ↔^^æãÁ~~*ÁÖÓNÞÁNSÖQÓÁOŠSÚÞŠQQÓÞÁâæ→~}ÁEÐÁ
ÁÁÐà→~á\ÁP*ÁKÁ€GIÌÁ
ÁÁÁÐÁ|ÁKÁÇãæàÁÁÉÁUæã{~N^&DEÁP*ÌÁ
ÁÁÁæÁKÁãæàÁÁÉÁUæã{~N^&ÌÁ
ÁÁÁ|ÁKÁÉÇäÐ´DE|FÁÉÁÇáÐ´DEæÁÉÁÇâÐ´DEæFÌÁÁ
ÁÁcÁ
cÁÁ
Á Á
ÐEÁÚåÞæàÊÁÚåÞæàÁFÊÁæÊÁæFÁÁãæÁ*ãæbæ^\ÊÁâ|\Á^~\Á|bæäÁ↔^^Á\å↔bÁæãæ↑~ÈÁ Á

Ò~}æ{æãÊÁ\åæ]Á}↔→→ÁâæÁ^æ´æbbáã]ÁÇá\ÁáÁ↑↔^↔↑|↑DÁ}åæ^Á\åæÁ´~^\ã~→→æãbÁ}↔→→ÁâæÁ
↔↑*→æ↑æ^\æäÈÁEÐÁ

Controller Discretization Matlab Code:

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P*ÁKÁËÎÌÁ
áÁKÁG€ÌÁ
âÁKÁGIÌÁ
PŽFÁKÁËFÈGĞÌÁ
\á|ÁKÁ€È€GIJÌÁ
ÁÁ
bÁKÁ\àÇCbCDÌÁ
OÁKÁP*EÇÇbÉáDÐÇbÉâDDÌÁ
ŞÁKÁPŽFÁÐÁÇbEÇ\á|EbÁÉÁFDDÌÁ
ÖÁKÁŞEOÁÐÁÇFÁÉÁŞEODÌÁ
ÁÁ
â}ÁKÁâá^ä}↔ä\àÇÖDÌÁ
â}ÁKÁÇâ}ÁÐÁÇGE*↔DDÌÁ
ÁÁ
bá↑*→æŽàãæ@ÁKÁGIEâ}ÌÁ
bá↑*→æŽ*æã↔~äÁKÁFÐbá↑*→æŽàãæ@ÌÁ
ÁÁ
OŽäÁKÁ´GäÁÇOÊÁbá↑*→æŽ*æã↔~äÊÁC\|b\↔^CDÌÁ
Y^|↑ÊÁäæ^ŸÁKÁ\àäá\áÇOŽäÊÁC{CDÌÁ
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cRIO-9076_RT.vi block diagram: