Lesson 2.3 ~ Tables and Graphs of Proportional Relationships

At a Glance

I can recognize proportional relationships in tables and graphs.

Standards Overview

Focus Content Standard(s): 7.RP.A.2a,b,d (Major)
Focus Math Practice Standard: SMP3

Materials

Optional: Graph paper (1st quadrant), rulers, color tiles, colored pencils, white copy paper

Starter Choice Board Overview

Storyboard: Solve a proportion (7.RP.A.3)
Building Blocks: Graph in four quadrants (6.NS.C.6)
Blast from the Past: Analyze a division error (4.MD.A.2)
Fluency Board Skills: Solve one-step equations, add and subtract decimals, find the volume of rectangular prisms

Lesson Planning Overview

Minutes	Day 1
5-10	**Starter Choice Board:** BB or BftP
15-20	**Explore! Activity:** "What Makes Me Proportional?"
15-20	**Lesson Presentation**
HW	**Practice:** Student Lesson or Online Practice

Minutes	Day 2
5-10	**Starter Choice Board:** Storyboard
20	**Teacher Gem:** ASN, CTL or TT
10	**Exit Card or Target Tracker**
HW	**Practice:** Leveled Practice

Learning Outcomes

All students should be able to determine whether a graph or table represents a proportional relationship.

Exit Card

1. Determine whether or not the table models a proportional relationship. If so, give the constant of proportionality.

x	y
0	0
1	0.5
2	1
3	1.5
4	2

2. Sketch a graph of a proportional relationship. What are the two things needed on any proportional relationship graph?

Most students should be able to solve problems arising in contextual situations involving proportional relationships.

Example: Student Lesson Exercise #8
Kirsten walked 4 miles per hour for 5 hours to train for an upcoming marathon walk.
a. Copy and complete the table below.

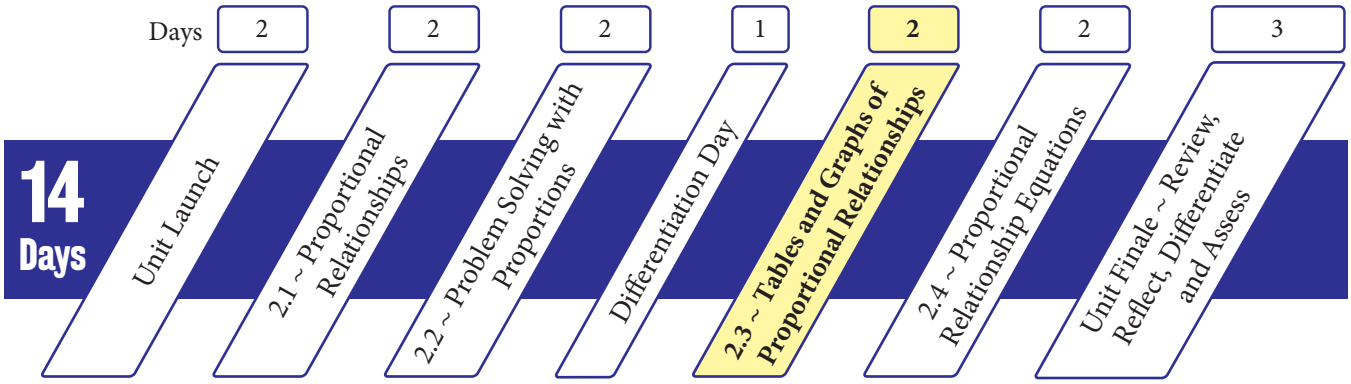
Number of hours Kirsten walked, x	0	1	2	3	4	5
Number of miles Kirsten walked, y						

b. Graph the ordered pairs in the table. Does it make sense to connect the points with a line? Explain your reasoning.
c. Does this situation model a proportional relationship? Explain your reasoning.

Lesson 2.3 ~ Tables and Graphs of Proportional Relationships

Deep Dive

I can recognize proportional relationships in tables and graphs.



Focus Content Standards

7.RP.A.2a, b, d (Major): Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Lesson Planning Guidance: Day 1

Starter Choice Board: Building Blocks or Blast from the Past (5-10 minutes)

In this lesson, the "Building Blocks" task asks students to access background knowledge on graphing in four quadrants. Use this activity if many of your students need support in recalling this skill. Consider using Expert Crayons to have students move around the room supporting each other. Choose the Starter Choice Board's "Blast from the Past" task to give students an opportunity to utilize problem solving skills involving analyzing a division error.

Explore! Activity: "What Makes Me Proportional?" (20-25 minutes)

"What Makes Me Proportional?" gives students the opportunity to discover how proportional relationships are represented in tables and graphs. First, students substitute values into given equations and look at the ratio of y to x to determine which table shows a proportional relationship. Next, students graph the input and output values as ordered pairs, connecting back to Grade 6 standards for graphing, and look for similarities and differences between the graphs to make a hypothesis about the two qualities of a graph of a proportional relationship.

Implementation Option #1: Assign students one of the four tables in Step 1 for them to complete individually (or consider modeling one and then assigning the others to individuals in groups of three). Have students continue with their table in Steps 2-3, taking time to check their team's work. Groups can then share their graphs and discuss Step 4 with their team.

Implementation Option #2: Have partner sets pick A, B, C or D and complete their part throughout Steps 1-3. Once they have worked through one equation and confirmed it was correct, assign them a different input-output table in Step 1 for them to complete (repeating Steps 1-3 again). Display the graphs for all four tables (answers for Step 3) and have a class discussion using the prompt in Step 4.

Lesson Presentation (15-20 minutes)

Have students utilize their interactive textbooks or composition notebooks to participate in guided note taking using the Lesson Presentation. Have students attempt Extra Examples with partners, in small groups or independently. Use "Communication Break" slides as opportunities for meaningful discourse.

Communication Break – Think, Ink, Pair, Square: Use the prompt "How might you be able to tell if a table of values represents a proportional relationship?" Have students think and write independently before joining with a partner to share. Then have two partner sets join together. Ask one group to start and the other group respond using the sentence stems provided.

Communication Break – Heads Together: Using Example 3, have students put pencils down and look at the question. Consider reading it together. Ask partner sets or small groups of students to examine the question and think about how it is similar or different to what they have already done in the lesson/unit and how that will affect solving the problem. Have students use the sentence stems to share out in groups and full class, if desired.

Independent Practice: Student Lesson Exercises or Online Practice

Have students complete all exercises from the first three sections of the Student Lesson: "Practice My Skills," "Reason and Communicate," and "Apply to the World Around Me." Differentiate based on student need by reducing the exercise set to odd numbers only or extending by only assigning the "Gem" exercises to specific students. If time allows, consider using the Spiral Review as an additional component. If choosing the Online Practice, consider using the Online Practice template for students to record their work and reflect on their results on either (or both) the Online Practice or Online Challenge Practice.

Lesson Planning Guidance: Day 2

Starter Choice Board: Storyboard Starter (5-10 minutes)

In this Storyboard Starter, students solve a proportion to convert measurements. Consider having students read the Storyboard frame and have silent think time for 1-2 minutes before working with table partners. Another option is to have one or two students in each group read the characters' statements and then work together to answer the prompt.

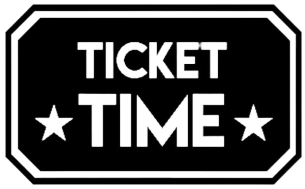
Teacher Gem: Always Sometimes Never, Climb the Ladder or Ticket Time (20 minutes)

Video Instructions

Best to use when students need deeper understanding of the vocabulary and concepts introduced in the lesson.

Video Instructions

Best to use when many students are ready to progress to higher depths of knowledge incrementally using the four ladders.



Video Instructions

Best to use when wanting to provide immediate feedback and gather classroom data around lower depth of knowledge questions based on the lesson's learning target.

Exit Card and Target Tracker (10 minutes)

Use the pre-printed Exit Card or have students use their own paper with the last slide of the Lesson Presentation. Have students update their Target Tracker using their Exit Card data and their Teacher Gem experience. Use the Exit Card data to provide guidance for Leveled Practice assignments or for an upcoming Differentiation Day.

Independent Practice: Leveled Practice

Hand out the Proficient and Challenge Leveled Practice copied front to back. Have students select the side they feel would be appropriate practice based on their self-reflection or assign specific students to specific sides based on information you have gathered during the lesson. Make answers available, if desired, by printing the answers and posting them in the room or posting them to your online platform so students have access to feedback as they work.

Formative Assessment Guidance

Many components in the EdGems Math program can be used to provide formative assessment data for teachers and students. The recommendations below provide suggested components for this lesson and guidance for using this data to differentiate instruction.

Option 1 – Student Lesson Exercises: Have students complete and self-correct the odd exercises in the "Practice My Skills" section of the Student Lesson. Students who get less than two of the items correct may need to join a small group for the remainder of the class or on an upcoming Differentiation Day. During that session, consider using the Tiered or Proficient Leveled Practice on paper or individual white boards.

Option 2 – Climb the Ladder: If the Climb the Ladder Teacher Gem was not used as a collaborative activity, have students complete Ladder 2 individually. If students are not able to complete 2 of the four questions correctly, move them back to Ladder 1 to complete in a small group with the teacher or an "Expert." If students were able to answer at least 2 of the items correctly, have them work independently or with a partner to complete Ladders 3 and 4 (giving both to them at the same time).

Option 3 – Tiered Leveled Practice: Have students complete problem #s 1, 3, 6, 9 on their own. Once completed, have students self-correct using answers projected on the white board or screen (using the answer key). Students would then complete the Target Tracker for this lesson and determine if they should join the teacher in a small group to complete the remainder of the Tiered Leveled Practice or if they could move to the Student Lesson exercises or the Proficient Leveled Practice.

Focus Math Practice

In this lesson, students will have many opportunities to construct viable arguments and critique the reasoning of others (SMP3). Have students use the Mathematical Practices Tracker to reflect on situations where they exhibited this practice (i.e., Explore! activity, Always Sometimes Never, Student Lesson Exercises #4-7).

I can explain, justify and critique the reasoning of myself and others.

I explained my thinking and justified my strategy.	I listened to different ways of thinking about a problem.	I helped someone else improve their own thinking.

Math Practices: Teacher and Student Moves

Math Practice	Teacher Moves	Student Moves
SMP1 *Make sense of problems and persevere in solving them.*	Use the Teacher Gem activities to develop students' abilities to make sense of and identify proportional relationships in tables and graphs, calculate constants of proportionality, and model proportional relationships using different representations.	During each Teacher Gem activity, make sense of each question, especially questions with multiple parts. Use feedback to improve models and calculations.
SMP2 *Reason abstractly and quantitatively.*	As students work through Student Lesson Exercises# 4-7, instruct them to relate their explanations to real-world situations. Ask the students to also discuss how each situation is or is not an example of a proportional relationship and how you know.	As you work through Student Lesson Exercises #4-7, relate each one back to a real-world situation. Explain how the numerical values (when given) help you understand the situation. Explain how each situation is or is not an example of a proportional relationship and how you know.
SMP3 *Construct viable arguments and critique the reasoning of others.*	Show students examples and non-examples of graphs of proportional relationships and ask students to make conjectures about which features of the graph to use to identify whether or not it represents a proportional relationship. Ask them to use their ideas to draw a new graph showing a proportional relationship.	When looking at examples and non-examples of graphs of proportional relationships, describe qualities of the graphs to a partner. Work together to determine the characteristics of graphs of proportional relationships.
SMP5 *Use appropriate tools strategically.*	Provide graph paper, rulers, color tiles, color pencils, blank paper, etc. Instruct students to select the appropriate tools and explain their choices prior to beginning a task.	During the lesson, choose tools from the provided materials. Explain why you chose the tools, and how they would help you to solve the problems. After solving each problem, explain how the tools helped you to solve the problems.
SMP6 *Attend to precision.*	Remind students to calculate the constant of proportionality using $\frac{y}{x}$ rather than $\frac{x}{y}$. Develop this understanding by connecting to independent and dependent variables as well as unit rates (i.e., dollars per hour rather than hours per dollar).	Calculate the constant of proportionality using the ratio of y to x. Complete the input-output tables and graphs in the Explore! activity with a degree of precision.
SMP7 *Look for and make use of structure.*	Have students compare and contrast the situations in Student Lesson Exercises #8-9 in order to recognize situations that represent a proportional relationship. Ask students to modify Student Lesson Exercise #9 so that it is proportional.	Compare the situations in Student Lesson Exercises #8-9. Identify which situation is a proportional relationship. Modify the context of the non-proportional relationship so that it is proportional.
SMP8 *Look for and express regularity in repeated reasoning.*	Use the Explore! activity and Example 1 to show students how to look at the pattern of ratios in the form $\frac{y}{x}$ in a table to determine whether or not the table represents a proportional relationship.	In the Explore! activity look for patterns (or lack of patterns) in the graphs to make a hypothesis about the qualities of a graph of a proportional relationship.

Supports for Students with Learning and Language Differences

Common Misconceptions

Some students have difficulty finding the unit rate on a graph (i.e., they may not remember if the unit rate is the (1, y) or (x, 1) point). Teachers may want to reinforce unit rate that can be recited by students (i.e, one movie ticket for $7) to help them recall that the x, which is first in a coordinate pair, is the 1; and the y (the 7 dollars) is the unit rate.

When finding points on a line that represents a real-world proportional relationship, students may think that the line stops at the origin. The teacher should show that the line continues into Quadrant III but that the points are not appropriate for some real world situations (i.e, in a proportional relationship between the number of homework helpers and the number of students in a class, it does not make sense to have a negative number of homework helpers).

While graphing, students may need to be reminded that the same types of quantities need to be graphed on the same axis. For example, when checking to determine if 5 cupcakes for $12.50 is proportional to 25 cupcakes for $62.50, the number of cupcakes must be represented on the same axis and the dollar amounts must be on the other axis. Ensure students are using graph paper or graphing calculators for all graphing. Remind them to label the axes.

Mathematical Language Routines

Designed by Stanford Graduate School of Education's Understanding Language, mathematical language routines are structured but adaptable formats designed to help teachers "amplify, assess and develop students' language" in math class. Three language routines specific to this lesson are given below.

MLR 1 – Stronger & Clearer Each Time: During the Teacher Gem activity Always Sometimes Never, instruct students to add examples and counterexamples on their evidence cards. Groups then exchange their evidence cards with 2 -3 other groups to get feedback on how to improve their explanations. When students receive their evidence cards back with the feedback, they revise their explanations. While groups are providing feedback, display the following questions to help them think about what types of feedback will be helpful:

- Did the group answer the question?
- Did the group provide examples and counterexamples to support their answer?
- Did the group justify their reasoning, referencing the examples and counterexamples?
- How can the group make their explanations stronger?

MLR 3 – Critique, Correct & Clarify: After the lesson, display the incorrect statement:

"In order to determine if a relationship is a proportional relationship, you only need to look at the graph of the relationship. If the graph of the relationship passes through the origin, it is a proportional relationship."

Students work with a partner to:

- identify the error(s) in the statement
- provide an example to support the statement and a counterexample to disprove the statement
- revise the statement
- present the revised statement to the class or small group

MLR 5 – Co-Craft Questions: Before starting Student Lesson Example #1, display the table about the hours babysitting and dollars earned.

Allow students 5 - 10 minutes to work with a partner to create mathematical questions based on the displayed information. Post the questions so students can compare their questions to the questions in the text. If time allows, choose questions as a class to solve.

Hours Babysitting, x	Dollars Earned, y
0	0
1	10
2	20
3	30
4	40
5	50
6	60