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PORTFOLIO SELECTION: REAL OR NOMINAL RETURNS?

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ABSTRACT

This study investigates the extent to which relying on nominal versus real returns affects the outcome of the portfolio selection problem. In other words, the goal is to explore if, for a given set of assets, and a prespecified risk budget, maximizing the portfolio nominal versus real return yielded similar asset allocation percentages. To this end, we rely on actual returns data for several asset classes, spanning a 20-year period (2004-2023). And to capture the nuances of different market regimes, the analysis is based on: (i) a sequence of sixteen overlapping 5-year windows; combined with (ii) five levels of risk constraints (defined by a CVaR limit) for each 5-year window. Therefore, all in all, 80 optimization (portfolio selection) problems are solved. The outcome is clear: both approaches, broadly speaking, result in different asset allocations. Moreover, in 20% of the cases, the differences in asset allocations (as measured by the normalized Euclidean difference) exceed 10%, which, in turn, translates into substantial differences in percentage allocations at the asset level. Most of the differences in asset allocations cann be explained by the fact that the nominal and real returns distributions of the relevant asset classes, as measured by the Kolmogorov-Smirnov test, are different. In short, the results suggest that before tackling the portfolio selection problem, one should make an assessment of the potential impact of inflation; that is, examine the difference between the real and nominal returns distributions. In summary, given that a rational investor should only be concerned about real (not nominal) returns, our results make a strong case that all portfolio selection problems should be formulated based on real returns.

Ignoring inflation (that is, casting the problem based on nominal returns) can lead to misleading, that is, suboptimal, asset allocations.

Keywords: portfolio selection; portfolio optimization; asset allocation; inflation.

BACKGROUND AND PREVIOUS WORK

The conceptual framework of the portfolio selection problem has remained unaltered since Markowitz's seminal work (Markowitz, 1952). It reduces, given a set of admissible investment options, to finding the weights (asset allocations) that maximize the return subject to a prespecified risk limit. Markowitz's paper triggered a tremendous research interest in the then nascent field of modern portfolio theory, an interest which remains vigorous to this day. The focus of these research efforts has been quite wide. The emphasis, however, has been on proposing algorithms to either improve the out-of-sample performance of the resulting portfolios, or, developing better—more stable or efficient—formulations of the associate optimization problem. More recently, attention has shifted to incorporating AI methods, often combined with the use of synthetic data. Surprisingly, a missing topic in these efforts has been the relevance of formulating the portfolio selection problem in terms of nominal versus real returns. This distinction is crucial for this choice can directly influence the resulting asset allocations. In practice, most financial data (e.g., returns data, index levels) are reported in nominal terms. And we can add, that at least recently and in most developed markets, inflation has been low. This might explain, although not necessarily justified, the lack of attention to this issue. Nevertheless, the fact remains that investors, at least rational investors, should only care about real returns.

The academic literature has largely neglected this topic. Papers dealing with portfolio selection are typically formulated based on nominal returns (although in general they are simply refer to as "returns", that is, not even acknowledging the existence of two different types of returns). And the potential differences in outcomes depending on which returns are used when casting the optimization problem is never discussed or even considered. Additionally, financial textbooks also introduce the portfolio selection problem with indifference to the real- versus nominal-returns issue. The exception is a 1975 study by Nahum Biger.

This author, to the best out knowledge, is the only researcher who has investigated whether asset allocation weights could differ, depending on whether one relies on real or nominal returns when formulating the portfolio optimization problem (Biger, 1975). His study, unfortunately, was based on a small dataset (stock returns from the 1950-1954 period). He stated rather tentatively, in reference to the asset allocation weights, that "the similarity [in asset weights] is less pronounced when lower expected returns are considered." But he did not attempt to elaborate any further on the reasons behind the asset allocation weights discrepancies detected for lower returns. He also computed the correlation matrices using both real and nominal returns, and although both are remarkably similar (Tables 2 and 3 in his paper), he somewhat inexplicably concluded that they were very different.

Two other authors, Solkik (1978) and Manaster (1979), looked at a tangentially related problem: whether a portfolio that is nominal efficient could also be real efficient. Their analyses were based on assumptions that now are known to be unrealistic (e.g., normality of asset returns, validity of the CAPM), but did not offer any insights related to the asset weights determined with either set of returns.

To sum up, as extraordinary as this could seem, in the last seventy years or so there has been no attention paid to this issue, namely, the implications of using real versus nominal returns when formulating the portfolio selection problem. Thus, the goal of this study is twofold: (i) to investigate the extent to which asset allocation weights might differ depending on whether one formulates the portfolio selection problem in terms of real or nominal returns; and (ii) if there are indeed important differences in asset allocations, identify the conditions under which such differences are more salient.

PROBLEM STATEMENT

Assume that we have N asset classes (each described by a suitable index), M (monthly) periods, and a risk budget Λ, controlled via the CVaR. We have selected the CVaR as the risk metric, in line with recent advances in financial engineering, since the CVaR focuses on losses, instead of the standard deviation of returns which measures dispersion. We further assume that we have relevant returns data and we denote as $r_{p,q}$ the return of asset class p in period q. Finally, let $\omega \in \mathbb{R}^N$ be the vector of assets weights, and, $\hat{r} \in \mathbb{R}^N$ be the vector of expected asset returns. The components of the vector of expected assets returns, $\hat{r}$, can be estimated as

$$\widehat{r_p} = \frac{1}{M} \sum_{q=1}^{M} r_{p,q}$$

and thus, the expected portfolio return, $\bar{R}$, can be expressed as

$$\bar{R} = \omega^T \hat{r}$$

The portfolio optimization problem then becomes

$$\text{Maximize } \bar{R}$$

subject to

$$CVaR_{\alpha}(\omega) \leq \Lambda$$

with

$$\sum_{p=1}^{N} \omega_p = 1$$

and

$$\omega_p \geq 0 \qquad \forall\ p \in (1, .., N).$$

$CVaR_\alpha(\omega)$ refers to the CVaR estimated based on the worst $(1-\alpha)\%$ loss scenarios. Notice that the preceding formulation is generic: valid regardless of the type of returns used.

The purpose of this study is to explore how the solution to the optimization problem, that is, the asset allocation vector $\Omega = \{\omega_1, \dots, \omega_N\}^T$ that maximizes the return, could vary depending on whether one relies on real or nominal returns to formulate such problem.

THE DATA

We considered five asset classes, namely, gold, investment grade (IG) corporate bonds, high yield (HY) bonds, global stocks, and short-term (ST) Treasurys, each described by a suitable index. The selected time frame was January 2004-December 2023; this period is long enough to capture different market regimes and spans two major crises: the subprime (2007-2008) crisis and the COVID-19 (2020-2022) crisis. The data were downloaded from the Bloomberg, Yahoo Finance, and the St. Louis Federal Reserve websites. Monthly real and nominal returns were calculated based on these indices and the U.S. CPI as reported by the U.S. Bureau of Labor Statistics.

METHOD OF ANALYSIS

To assess the differences between portfolios optimized using nominal returns and those optimized using real returns, we conducted an analysis based on sixteen rolling 5-year windows, from [2004-2008] to [2019-2023]. And in each window, we solved five paired optimization problems based on the scheme described below. (Note: the motivation for using several time-windows, instead of collapsing all the data in one "big" window, (2004-2023), is to capture better the nuances of different market regimes.)

For each window we proceed as follows.

- We compute the CVaR(90%) for each asset class, based on nominal returns, to identify the overall maximum and minimum CVaR value. Then, we segment the interval defined by this range using five equidistant points. Each point corresponds to a specific CVaR value: $\Lambda_1, \dots, \Lambda_5$.

- We solve, for each Λ_i $(i = 1, \dots, 5)$, the corresponding optimization problem, based on nominal returns. We designate the resulting asset allocation as Ω_N.

- We calculate the CVaR defined by this asset allocation (Ω_N), using real returns. We designate this CVaR as Λ_R.

- We solve a new optimization problem, based now on real returns, but using Λ_R to specify the risk constraint. We designate the resulting asset allocation as Ω_R.

- We compare these two asset allocations (Ω_N and Ω_R), by means of the Euclidean difference between these two vectors, normalized by $||\Omega_N||$. We denote the difference (expressed as a percentage) as Δ.

In brief, we conducted 80 asset allocation comparisons: 16 (time windows) x 5 (five paired optimizations for each window) = 80; each comparison is associated with its corresponding Δ.

RESULTS AND DISCUSSION

The results are summarized in Table 1. They show that in 16 out of the 80 cases (20%), the discrepancies in asset allocations (weights), as measured by Δ, exceed 10%. This level of discrepancy in asset allocations is not trivial, as Table 2, which is more specific, indicates. This table shows the asset allocation weights (ω's), for a few selected cases over a wide range of Δ-values. For example, in the case of the [2008-2012] window and Δ = 48.4% (which corresponds to the maximum discrepancy between Ω_N and Ω_R considering all 80 cases analyzed) the differences in asset allocation levels are very salient. In fact, the two sets of Ω's define two completely different portfolios. Perhaps more relevant, even in the case where Δ is slightly lower than 10%, the differences in asset allocations are important. Consider the [2007-2011] case with Δ = 8.1%. No asset manager would be indifferent between having almost 4% in IG bonds and not holding them at all; or, between a 31.0% versus a 26.5% position in HY bonds. All other cases in which Δ > 10% (not shown in Table 2, but available upon request) reveal situations analogous to those displayed in Table 2, namely, important differences in asset allocations, which, are roughly correlated with the value of Δ.

Table 1: Summary of Portfolio Optimization Results for All Time Windows

[2004 - 2008]	CVaR Nominal	8.05%	6.60%	5.15%	3.70%	2.25%
Inflation 2.68%	Δ	0.00%	0.00%	0.00%	0.00%	**16.03%**
[2005 - 2009]	CVaR Nominal	9.23%	7.55%	5.87%	4.19%	2.51%
Inflation 2.57%	Δ	0.00%	0.00%	0.00%	**18.04%**	**21.45%**
[2006 - 2010]	CVaR Nominal	9.49%	7.75%	6.01%	4.26%	2.52%
Inflation 2.19%	Δ	0.00%	0.00%	0.00%	**12.11%**	**27.60%**
[2007 - 2011]	CVaR Nominal	9.58%	7.81%	6.04%	4.27%	2.51%
Inflation 2.27%	Δ	0.00%	7.96%	**26.95%**	**19.04%**	**10.81%**
[2008 - 2012]	CVaR Nominal	9.57%	7.80%	6.03%	4.26%	2.49%
Inflation 1.80%	Δ	0.00%	0.00%	0.00%	**28.91%**	**48.38%**
[2009 - 2013]	CVaR Nominal	7.72%	6.29%	4.86%	3.43%	1.99%
Inflation 2.08%	Δ	0.00%	0.00%	0.00%	0.00%	0.79%
[2010 - 2014]	CVaR Nominal	7.51%	6.06%	4.62%	3.18%	1.74%
Inflation 1.69%	Δ	0.00%	0.00%	0.00%	0.00%	0.00%
[2011 - 2015]	CVaR Nominal	7.64%	6.17%	4.69%	3.22%	1.75%
Inflation 1.54%	Δ	0.00%	0.00%	0.00%	0.00%	**12.50%**
[2012 - 2016]	CVaR Nominal	6.58%	5.33%	4.09%	2.84%	1.59%
Inflation 1.36%	Δ	0.00%	0.00%	0.00%	0.00%	8.06%
[2013 - 2017]	CVaR Nominal	6.55%	5.31%	4.07%	2.83%	1.60%
Inflation 1.43%	Δ	0.00%	0.00%	0.00%	4.60%	6.80%
[2014 - 2018]	CVaR Nominal	5.32%	4.33%	3.34%	2.35%	1.36%
Inflation 1.52%	Δ	0.00%	0.00%	0.00%	0.00%	**19.03%**
[2015 - 2019]	CVaR Nominal	5.26%	4.28%	3.30%	2.32%	1.34%
Inflation 1.82%	Δ	0.00%	0.00%	0.00%	3.13%	**25.17%**
[2016 - 2020]	CVaR Nominal	6.78%	5.49%	4.21%	2.92%	1.63%
Inflation 1.95%	Δ	0.00%	0.00%	0.00%	0.66%	8.71%
[2017 - 2021]	CVaR Nominal	6.52%	5.28%	4.04%	2.80%	1.56%
Inflation 2.94%	Δ	0.00%	0.00%	0.00%	2.02%	**10.03%**
[2018 - 2022]	CVaR Nominal	7.87%	6.48%	5.09%	3.70%	2.32%
Inflation 3.81%	Δ	0.00%	0.00%	0.00%	4.26%	9.02%
[2019 - 2023]	CVaR Nominal	7.63%	6.31%	4.98%	3.66%	2.34%
Inflation 4.10%	Δ	0.00%	0.00%	0.00%	**20.36%**	**26.34%**

Note: For each 5-year window, the table reports the five nominal CVaR levels used as risk constraint in the optimization, the corresponding average annual inflation rate for that window, and Δ (expressed as a percentage) which captures the difference between the nominal and real optimal asset allocations.

Table 2: Asset Allocation Weights for Selected Cases

Time Window	2007 - 2011		2008 - 2012		2008 - 2012		2017 - 2021	
Target CVaR	7.81%	7.71%	4.26%	4.13%	2.49%	2.50%	1.56%	1.97%
Asset Class	Nominal	Real	Nominal	Real	Nominal	Real	Nominal	Real
Gold	69.0%	69.8%	25.9%	27.5%	14.8%	14.6%	26.9%	26.4%
IG Corp Bonds	0.0%	3.8%	19.4%	31.9%	0.4%	21.1%	0.0%	0.0%
US HY Bonds	31.0%	26.5%	42.8%	35.9%	31.5%	24.7%	0.0%	4.4%
Global Stocks	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	17.9%	18.3%
ST Treasurys	0.0%	0.0%	12.0%	4.7%	53.4%	39.7%	55.2%	51.0%
Δ	**8.1%**		**28.9%**		**48.4%**		**10.0%**	

In short, whether one formulates the optimization problem based on nominal or real returns (that is, neglecting or not the effects of inflation) can have a substantial impact on the resulting asset allocation weights. Therefore, it is worth investigating under what conditions the asset allocations (Ω_N and Ω_R) tend to differ more, or, alternatively, what are the drivers behind the differences.

Clearly, at the root of the differences in the Ω's, is the difference between the distributions of the real and nominal returns. To asses the difference between these two distributions, we relied on the Kolmogorov-Smirnov (KS) test. We performed this test for each asset class and for each of the 5-year windows. Evidently, a difference in the distribution of real and nominal returns is only relevant to the extent that the ω associated with that asset class is non-zero. Therefore, to capture better the potential effect of the difference between the two return distributions on the resulting asset allocations (in essence, Δ), we defined a new metric, π_{KS}, as follows:

$$\pi_{KS} = \frac{A}{B}$$

where, B is the number of non-zero ω's associated with that specific portfolio allocation (when relying on the nominal returns), hence, B can only take values between 1 and 5. Then, we assign an index (1 or 0) to each asset class present in this portfolio allocation, depending on whether the KS test was significant at the 90% confidence-level or not. A is the sum of such indices. Thus, a value of π_{KS} equal to zero implies that for the asset classes present in that portfolio the distribution of real and nominal returns are not different; on the other hand, a value of π_{KS} equal to one implies that for all relevant asset classes the distributions are different. In short, the π_{KS} metric (note that $0 \leq \pi_{KS} \leq 1$) represents an effort to capture the degree to which the real and nominal return distributions, considering only the asset classes present on a given portfolio, are different.

Additionally, suspecting that deviations from normality (due to the presence of "heavy tails") might magnify the effect of inflation on the returns distributions, we employed a Jarque-Bera (JB) test applied to the nominal returns distribution. And we defined an additional metric, π_{JB}, analogous to the π_{KS} metric, but based on the JB test.

Recall also that Biger (1975) had suggested that differences in asset allocations were associated with lower returns. Table 1, at least visually, seems to validate this view as higher values of Δ are more prevalent on the right-hand side the table, which, is associated with lower CVaR-values (and therefore, lower returns). Intuitively, this seems reasonable: each 5-year window is characterized by a specific inflation level. However, in relative terms, the difference between nominal and real returns is magnified in the case of lower nominal returns.

To explore the potential explanatory power of these variables we regressed the difference in asset allocations (Δ) on $CVaR_{Nominal}$, π_{KS} and π_{JB}. Table 3 shows the results.

Table 3: Regression Results: Several Models

Variables	(1) Δ	(2) Δ	(3) Δ	(4) Δ
$CVaR_{Nominal}$	-1.822***			
	(0.439)			
π_{JB}		0.378***		0.171**
		(0.065)		(0.079)
π_{KS}			0.666***	0.494***
			(0.095)	(0.122)
Constant	0.137***	-0.002	0.015	0.000
	(0.023)	(0.013)	(0.010)	(0.012)
Observations	80	80	80	80
R-squared	0.181	0.299	0.387	0.423

Note: The table shows the results of regressing Δ on different choices of independent (explanatory) variables, namely, nominal CVaR, π_{JB} (Jarque-Bera test for normality), and π_{KS} (Kolmogorov-Smirnov test for distribution differences). Standard errors in parentheses. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Not surprisingly, the highest explanatory power ($R^2 = 0.39$) is associated with π_{KS}. That is, with the extent to which both (real and nominal) distributions are different. Adding π_{JB} (deviations from normality) improves marginally the model ($R^2 = 0.42$); the π_{KS}-coefficient is roughly three times that of the π_{JB}. The $CVaR_{Nominal}$ and π_{KS} are highly correlated (almost 70%), and thus cannot be included in the same regression. Notice also the sign (negative) associated with $CVaR_{Nominal}$. Since, obviously, CVaR-values, that is, risk, and returns, move in tandem, this situation validates the comment made by Biger (1975): "…the similarity [in asset allocations] is less pronounced when lower expected returns are considered." To summarize, this simple regression analysis corroborates what one could have anticipated based on a theoretical reasoning: the differences in asset allocations are largely the result of differences in the return distributions of the assets that are most relevant in the optimization problem.

Two final observations. First, we did explore the possibility that differences in the correlation matrices (again, determined with real and nominal returns) could impact the differences in asset allocations (e.g., Δ). To investigate this effect (borrowing the key idea behind principal component analysis) we defined two variables: one variable based on the difference between the largest eigenvalues of the two matrices, and a second variable, based on the (cosine) difference between the two largest eigenvectors. However, regressing Δ on these variables proved that they had nil explanatory power.

And second, it is worth mentioning that of the five asset classes considered, the ST Treasurys represented the asset class in which the differences between real and nominal returns was the most severe. This is to some extent to be expected since short-term fixed-coupon bonds are very sensitive to inflation. In fact, their prices are largely dictated by inflation expectations. Moreover, removing from the set of feasible assets the ST Treasurys and running the optimization problem considering only the remaining four assets, reduced the number of scenarios with Δ > 10% to 4 (only 5% of the total). Again, this underscores the importance of the difference between the distribution of real and nominal returns on the asset allocation.

CONCLUSIONS

The results of this study show that the choice between nominal and real returns when casting the portfolio selection problem is consequential: in 20% of the cases analyzed the differences in asset allocations were substantial. In fact, in some cases, they resulted in entirely different portfolios. This finding emphasizes that neglecting the effects of inflation can materially affect the asset allocation choice.

Somehow expectedly, the degree of divergence between the distributions of real and nominal returns—as measured by the Kolmogorov-Smirnov test—explains most of the differences in asset allocations. Deviations from normality in the returns distributions also have an effect on the asset allocations, although it is less pronounced. Additionally, it is worth mentioning that differences in asset allocations are more likely to manifest in cases of lower than higher returns (e.g., lower CVaR values), a tendency already identified by Biger (1975).

Finally, assuming that asset allocations resulting from solving an optimization problem based on real versus nominal returns would be similar, is unwarranted. It seems prudent that before tackling the portfolio selection problem (especially when the feasible set includes short-term fixed-coupon securities) one should test the degree of similarity between the nominal and real return distributions. Therefore, incorporating inflation in the portfolio selection problem is a necessity, not an option. More to the point: one can easily argue that from a practical (as well as theoretical) standpoint, a rational investor should only care about real returns. Therefore, the most reasonable approach to tackle the portfolio selection problem is using real (and not nominal) returns. Relying on nominal returns can lead to suboptimal asset allocations.

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